

# **AN INTEGRATED APPROACH FOR ROBUST AIRLINE SCHEDULING, AIRCRAFT FLEETING AND ROUTING WITH CRUISE SPEED CONTROL**

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Hüseyin GÜRKAN

July, 2014

I certify that I have read this thesis and that in my opinion it is fully adequate,  
in scope and in quality, as a thesis for the degree of Master of Science.

---

Prof. Dr. M. Selim AKTÜRK(Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate,  
in scope and in quality, as a thesis for the degree of Master of Science.

---

Assoc. Prof. Dr. Sinan GÜREL(Co-Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate,  
in scope and in quality, as a thesis for the degree of Master of Science.

---

Assoc. Prof. Dr. Oya E. KARAŞAN

I certify that I have read this thesis and that in my opinion it is fully adequate,  
in scope and in quality, as a thesis for the degree of Master of Science.

---

Assist. Prof. Dr. Sakine BATUN

Approved for the Graduate School of Engineering and Science:

---

Prof. Dr. Levent Onural  
Director of the Graduate School of Engineering and Science

## ABSTRACT

# AN INTEGRATED APPROACH FOR ROBUST AIRLINE SCHEDULING, AIRCRAFT FLEETING AND ROUTING WITH CRUISE SPEED CONTROL

Hüseyin GÜRKAN

M.S. in Industrial Engineering

Supervisor: Prof. Dr. M. Selim AKTÜRK

Co-Supervisor: Assoc. Prof. Dr. Sinan GÜREL

July, 2014

To place emphasis on profound relations among airline schedule planning problems and to mitigate the effect of unexpected delays, we integrate robust schedule design, fleet assignment and aircraft routing problems within a daily planning horizon while passengers' connection service levels are ensured via chance constraints and maintenance requirements are satisfied. We propose a nonlinear mixed integer programming model. In the objective function, the cost functions due to fuel consumption and CO<sub>2</sub> emission cost involve nonlinearity. This nonlinearity is handled by second order conic reformulation. The key contribution of this study is to take into account the cruise time control for the first time in an integrated model of these three stages of airline operations. Changing cruise times of flights in an integrated model enables to construct a schedule to increase utilization of efficient aircraft and even to decrease the total number of aircraft needed while satisfying service level and maintenance requirements for aircraft fleet and routing. Besides, for the robust schedule design problem, it is possible to improve the solution since a routing decision could eliminate the necessity of inserting idle time or compressing cruise time. In addition, we propose two heuristic methods to solve large size problems faster than the integrated model. Eventually, computational results using real data obtained from a major U.S. carrier are presented to demonstrate potential profitability in applying the proposed solution methods.

*Keywords:* robust airline scheduling, aircraft fleet and routing, cruise time controllability, second order cone programming.

## ÖZET

# DAYANIKLI HAVAYOLU ÇİZELGELEME, FİLO TİPİ ATAMA VE UÇAK ROTALAMA PROBLEMLERİNE SEYİR SÜRESİ KONTROLÜ İLE BÜTÜNLEŞİK BİR YAKLAŞIM

Hüseyin GÜRKAN

Endüstri Mühendisliği, Yüksek Lisans

Tez Yöneticisi: Prof. Dr. M. Selim AKTÜRK

Eş-Tez Yöneticisi: Doç. Dr. Sinan GÜREL

Temmuz, 2014

Havayolu çizelgeleme problemleri arasındaki yoğun bağlantıyı dikkate almak ve beklenmeyen gecikmelerin etkilerini hafifletmek amacıyla, gürbüz havayolu çizelge tasarımı, filo tipi atama ve uçak rotalama problemleri günlük planlama çerçevesinde yolcuların bağlantı hizmet seviyelerini şans kısıtları ile garanti altına alarak ve bakım gereksinimlerini karşılayarak birleştirilmiştir. Karma tamsayılı doğrusal olmayan programlama formülasyonu geliştirilmiştir. Amaç fonksiyonundaki, yakıt tüketimi ve CO<sub>2</sub> salınımı maliyet fonksiyonları doğrusalsızlık içermektedir. Bu doğrusalsızlık ikinci derece konik reformülasyonlarla işlenmiştir. Bu çalışmanın en önemli katkısı, seyir süresi kontrolünün, ilk defa bu üç havayolu operasyonunun birleşiminde ele alınmasıdır. Uçuşlarının seyir sürelerini birleşik bir modelde değiştirmek, verimli uçakların kullanımı artıran bir çizelge geliştirilmesini, hatta aynı hizmet seviyesi ve bakım şartları için toplam ihtiyaç duyulan uçak sayısını düşürmeyi sağlamaktadır. Ayrıca, yeni bir rotalama kararı atıl zaman eklemesi ya da seyir süresi sıkıştırması gerekliliklerini ortadan kaldıracı için, gürbüz çizelge tasarım probleminde daha gelişmiş bir sonuç elde etmek mümkündür. Ek olarak, büyük ölçekli problemleri birleşik modelden daha hızlı çözebilmek için iki algoritma geliştirilmiştir. Son olarak, önerilen yöntemlerin karlılığını göstermek amacıyla ABD’li büyük bir havayolu şirketi tarafından yayımlanan verileri kullandığımız sayısal bir çalışmanın sonuçları sunulmuştur.

*Anahtar sözcükler:* dayanıklı havayolu çizelgeleme, filo tipi atama ve uçak rotalama, seyir süresi kontrolü, ikinci derece konik programlama.

## Acknowledgement

I would like to express my deepest gratitude to my advisor, Professor M. Selim Aktürk, for his excellent guidance, caring, patience, and providing me with an excellent atmosphere for doing research. The good advice, support and friendship of my co-advisor, Associate Professor Sinan Gürel, has been invaluable on both an academic and a personal level, for which I am extremely grateful. It is a privilege to work with them.

Due to their helpful discussions on critical points of this thesis, I would really appreciate Professor Hande Yaman, Özge Şafak and Uğur Arıkan.

I also would like to acknowledge the financial support of The Scientific and Technological Research Council of Turkey (TUBITAK) for the Graduate Study Scholarship Program they awarded.

Many thanks to my friends in graduate programs of industrial engineering department İrfan Mahmutogulları, Halenur Şahin, Okan Dükkancı, Bengisu Sert, Haşim Özlü, Meltem Peker, Ramez Kian, Esra Koca, Sinan Bayraktar, Gizem Özbaygın, Ece Demirci, Hatice Çalık, Başak Yazar and my officemates Nihal Berktaş, Merve Meraklı, Nil Karacaoğlu, Özüm Korkmaz, Burcu Tekin for their support which always keeps me motivated in really challenging conditions of M.S. study ...

I will always be grateful to my friends who welcome me to their home Onursal Bağırhan, Murat İplikçi and Ali Yılmaz for their invaluable friendship. Moreover, it was joyful to spend time during the M.S. study with Yunus Emre Kesim, Oğuz Çetin, Arif Usta, Serkan Pekçetin, Anıl Armağan, Fatih Çalışır and Merve Çalışır.

Eventually, my parents Nail and Aynur Gürkan and my little brother Onur Gürkan have given me eternal support, love and encouragement; they were always there cheering me up and stood by me through the good times and bad.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Motivation . . . . .	1
1.2	Contributions . . . . .	2
1.3	Overview . . . . .	4
<b>2</b>	<b>Literature Review</b>	<b>5</b>
2.1	Robustness in Airline Planning Process . . . . .	6
2.2	Airline Schedule Planning Problems . . . . .	7
2.2.1	Schedule Design Problem . . . . .	7
2.2.2	Fleet Assignment Problem . . . . .	8
2.2.3	Maintenance Routing . . . . .	9
2.3	Integrated Problems . . . . .	11
2.3.1	Schedule Design and Fleet Assignment . . . . .	11
2.3.2	Fleet Assignment and Aircraft Routing . . . . .	12
2.3.3	Schedule Design, Fleet Assignment and Aircraft Routing . . . . .	13

2.4	Second Order Cone Programming . . . . .	14
2.5	Cruise Speed Control versus Fuel Consumption and CO2 Emission	14
2.6	Summary . . . . .	15
<b>3</b>	<b>Problem Definition</b>	<b>16</b>
3.1	Distribution of Non-cruise Times . . . . .	17
3.1.1	Log-Laplace Distribution . . . . .	18
3.2	Fuel Consumption and CO2 Emission Cost . . . . .	19
3.3	Service Level . . . . .	20
3.4	Numerical Example . . . . .	20
3.5	Summary . . . . .	26
<b>4</b>	<b>Problem Formulation</b>	<b>28</b>
4.1	Mathematical Model . . . . .	30
4.1.1	Challenges for Solving the Model . . . . .	33
4.2	Reformulation of the Model . . . . .	34
4.2.1	Closed Form Expressions for the Chance Constraints . . .	35
4.2.2	Conic Representation of the Fuel Consumption and CO2 Emission Cost Functions . . . . .	35
4.2.3	Conic Reformulation of the Model . . . . .	37
4.3	Summary . . . . .	38
<b>5</b>	<b>Heuristic Algorithms</b>	<b>39</b>

5.1	Discretized Approximation and Cruise Speed Control Algorithm . . . . .	39
5.2	Multi-Stage Triplet Search Algorithm . . . . .	42
5.2.1	Numerical Example . . . . .	45
5.3	Summary . . . . .	48
<b>6</b>	<b>Computational Study</b>	<b>49</b>
6.1	Experimental Design . . . . .	49
6.2	Input Data . . . . .	51
6.3	Analysis on the Integrated Model . . . . .	55
6.3.1	The effect of cruise speed control . . . . .	56
6.4	Analysis on the Heuristic Methods . . . . .	58
6.4.1	Performance Analysis of Heuristic Methods . . . . .	59
6.4.2	Analysis on the structure of heuristic methods . . . . .	62
6.5	Summary . . . . .	64
<b>7</b>	<b>Conclusions and Future Works</b>	<b>66</b>
7.1	Summary of Thesis . . . . .	66
7.2	Future Works . . . . .	67
<b>A</b>	<b>Computational Results</b>	<b>74</b>
A.1	23 Flight Network . . . . .	74
A.2	35 Flight Network . . . . .	84



A.3 114 Flight Network . . . . .	92
----------------------------------	----

# List of Figures

3.1	Time space network of the published schedule . . . . .	22
3.2	Time space network of the proposed schedule . . . . .	24
3.3	The fuel & CO2 cost of flight 1438 with each aircraft . . . . .	25
3.4	The total cruise time in the published and the proposed schedules	26
5.1	Discretized approximation and cruise speed control algorithm . . .	42
5.2	Multi-Stage Triplet Search, Beam Size $b = 3$ , Depth Size $d = 5$ . .	46
5.3	Total costs of the solutions . . . . .	47
6.1	Effect of CSCM on fuel & CO2 cost and idle time cost . . . . .	63
6.2	Improvement over root node solution . . . . .	65

# List of Tables

2.1	Maintenance checks . . . . .	10
3.1	Published schedule . . . . .	21
3.2	Cost calculation for the published Schedule . . . . .	23
3.3	Proposed schedule . . . . .	23
3.4	Cost calculation for proposed schedule . . . . .	27
6.1	Factor values . . . . .	50
6.2	Published schedule for 114 flight network . . . . .	52
6.3	Problem size with different instances . . . . .	53
6.4	Aircraft parameters . . . . .	53
6.5	Aircraft type . . . . .	54
6.6	Congestion coefficients . . . . .	55
6.7	Cost improvement over the published schedule . . . . .	55
6.8	Total cost improvement of cruise speed control . . . . .	57
6.9	CPU time analysis . . . . .	58

6.10	Gap of heuristic methods over 23 flight network . . . . .	59
6.11	Improvement of heuristic methods over 35 flight network . . . . .	60
6.12	Improvement of heuristic methods over 114 flight network . . . . .	61
6.13	CPU time analysis . . . . .	61
A.1	Cost for the schedule generated by the integrated model . . . . .	74
A.2	Cost for the schedule generated by heuristic1 . . . . .	76
A.3	Cost for the schedule generated by heuristic2 . . . . .	77
A.4	Cost for the schedule generated by heuristic2 at the root node . . . . .	79
A.5	Cost for the published schedule . . . . .	80
A.6	CPU time . . . . .	82
A.7	Cost for the schedule generated by the integrated model in 5400 sec. . . . .	84
A.8	Cost for the schedule generated by heuristic1 . . . . .	86
A.9	Cost for the schedule generated by heuristic2 . . . . .	87
A.10	Cost for the schedule generated by heuristic2 at the root node . . . . .	89
A.11	CPU time . . . . .	90
A.12	Cost for the schedule generated by heuristic1 . . . . .	92
A.13	Cost for the schedule generated by heuristic2 . . . . .	93
A.14	Cost for the schedule generated by heuristic2 at the root node . . . . .	95
A.15	Cost for the published schedule . . . . .	97
A.16	CPU time . . . . .	99

# Chapter 1

## Introduction

The integrated robust airline scheduling, aircraft fleetling and routing problem is to develop a flight schedule, to assign aircraft fleet type to each flight and to generate routes for each aircraft simultaneously within a daily planning horizon for a given set of flights and a set of aircraft in an integrated manner while considering maintenance requirements and passengers' connection service levels. Due to its various considerations and numerous parameters, it is a challenging and complex problem. In this study, a mathematical model and two heuristic methods are developed and implemented in Java with a connection to a commercial solver, IBM ILOG CPLEX 12.6.

### 1.1 Motivation

Airline schedule planning process is to generate a schedule having the largest revenue under the consideration of fleet assignment, aircraft maintenance routing and crew assignment. Since it is a large and complex problem, it is often divided into subproblems and solved sequentially. However, most of the time this sequential approach causes suboptimal solutions due to the profound relations among these stages. In order to improve these suboptimal solutions, integrated models which consider combinations of these subproblems to compose are suggested.

Again the scope of the integration is limited by the tractability issue of the suggested models. Even if it is possible to solve a global airline schedule planning problem, during the implementation, still many disruptions are faced that result in operational delay. Therefore, two approaches are adopted to overcome these disruptions: robust planning and recovery models. The difference between robust planning and recovery models is the time when these models handle a disruption; while robust planning aims to construct a plan resilient to disruptions, recovery models try to suggest a new schedule soon after a disruption occurs. Therefore, it can be stated that for airline scheduling problem, an integrated model which considers robustness is of the essence.

## 1.2 Contributions

To the best of our knowledge, this is the first study in which cruise speed/time is controlled within the integrated robust schedule design, aircraft fleet and routing problem. Changing cruise time of flights in an integrated model enables to construct a schedule with flight sequences which are not considered previously due to fixed cruise speed/time such as a sequence with more flight legs or a sequence including two flight legs that cannot connect to each other. For two flights to be connected, performed by the same aircraft, there must be a gap between departure times of these flights. This gap is the sum of cruise time of the flight, required non-cruise time for the flight, turn around time and idle time. In other studies, the lower bound for this gap is taken as fixed, however cruise speed/time change enables to control this lower bound on the gap between departure times. By this means, in our study more flight connection alternatives could be generated. Due to having more alternatives on flight connections, it is possible to increase the utilization of efficient aircraft and decrease the cost of robustness. Moreover, in our study we present the second order conic reformulation of a non-linear mathematical model and make it solvable while proposing two heuristic algorithms for problems with larger instances.

The first contribution brought by our study is that aircraft utilization could be increased and even the total number of aircraft needed to cover a set of flights

could be decreased while ensuring service level and maintenance requirements. Due to having more alternatives on flight connections and compression of cruise time of flights, it is possible to increase the number of flights to be performed by an efficient aircraft. While this increase in the utilization of efficient aircraft could reduce the minimum number of required aircrafts to perform a set of flights, in addition the total cost of fuel consumption could be decreased.

The second is the robustness issue. Since we have more alternatives on flight connections, it is possible to generate better flight sequences in terms of robustness. For example, on a route having a flight with a great delay probability would require an intervention for the following flights to be performed on time while removing the problematic flight from that sequence could render that intervention unnecessary. Our study has more options to make these types of changes on routing decisions so changing routes could decrease the cost of robustness.

The third is the reformulation. We propose a non-linear mixed integer programming model. In order to solve this model analytically we tackled the non-linear cost components by representing them as second order conic inequalities. More information about conic programming can be found in Ben-Tal and Nemirovski [1] and Günlük and Linderoth [2]. We are able to solve a mixed integer second order conic programming formulation with a commercial solver, IBM ILOG CPLEX 12.6.

Lastly, in order to solve the large scale problems in a reasonable time, we propose two heuristic methods. The first one is discretized approximation and cruise speed control algorithm and the second one is multi-stage triplet search algorithm. In discretized approximation and cruise speed control algorithm, initially a mixed integer programming model, discretized approximation model, in which the cruise time is discretized, and then a nonlinear model, cruise speed control model, in which the cruise speed can take continuous values, are solved sequentially. In the multi-stage triplet search algorithm, a triplet refers to two consecutive flights and the aircraft which performs them. Briefly, in that algorithm, a search of triplets with high cost related to fuel and idle time is performed

over the schedules which are generated by the sequential solution of a mixed integer programming model, daily usage and spill costs model, and the nonlinear model, cruise speed control model.

## 1.3 Overview

In the next chapter, we present a detailed literature review regarding airline scheduling problems, cruise time controllability, fuel consumption of flights, methods to deal with the chance constraints and second order cone programming.

We elaborate the problem definition with the parameters considered in the problem in Chapter 3. Moreover, the major concepts in the problem such as distribution of non-cruise times, service level, fuel and CO<sub>2</sub> emission cost and maintenance feasibility are explained in detail. Finally, a numerical example is given to illustrate the frameworks of the problem.

In Chapter 4, we present the proposed mathematical model. Subsequently, conic representations of nonlinear objective function are explained. Lastly, we provide the conic reformulation of the model.

In Chapter 5, we introduce two heuristic methods. The first one is discretized approximation and cruise speed control algorithm and the second is multi-stage triplet search algorithm. Due to long solution time and numerical stability problems of the integrated model, these methods are proposed. In this chapter, the steps of the algorithms are elaborated in detail.

Chapter 6 is devoted to the computational study in which the performance of three proposed methods are compared. The numerical results which are obtained from two  $2^k$  full-factorial experimental design for three different sample data are presented. Afterwards, the methods are compared in terms of solution time and costs. Eventually, we conclude with future research extensions of the problem in the last chapter.



## Chapter 2

### Literature Review

Airline schedule planning process is to generate a schedule having the largest revenue under the consideration of fleet assignment, aircraft maintenance routing and crew assignment. Since it is a large and complex problem, it is often divided into subproblems and solved sequentially. Airline scheduling problems are divided into four stages, which are schedule design, fleet assignment, maintenance routing and crew assignment. Schedule design determines the flights to be flown and their departure times in consideration of the market demand and profitability. Fleet assignment models assign a particular fleet to each flight in the schedule by considering operational and spill costs. After a set of flights which will be covered by a particular fleet type are determined, maintenance routing problem, which is an aircraft routing model finding feasible routes in terms of maintenance for each aircraft in that fleet, is solved. As a last stage, crew assignment problem is solved for each aircraft on the corresponding flights. Each stage uses the output of the previous stage as an input, i.e., schedule design determines the flights to be flown and what will be the frequency, then fleet assignment problem takes these flights as an input. However, most of the time this sequential approach causes suboptimal solutions and sometimes in-feasibility due to the profound relations among these stages. In order to improve these suboptimal solutions, integrated models which consider combinations of these subproblems are suggested. Again the scope of the integration is limited by the tractability issue of the suggested

models. Even if it is possible to solve a global airline schedule planning problem, during the implementation, still many disruptions are faced that result in operational delay. Therefore, two approaches are adopted to overcome these disruptions: robust planning and recovery models. The difference between robust planning and recovery models is the time when these models handle a disruption; while robust planning aims to construct a plan resilient to disruptions, recovery models try to suggest a new schedule soon after a disruption occurs. For a recent survey which elaborates the problems of airline scheduling the study by Barnhart and Cohn [3] or Gopalan and Talluri [4] can be considered.

In this chapter, we present a literature review regarding robustness in airline planning process, airline scheduling problems and their integrations. Moreover, a literature review regarding second order cone programming and cruise time versus fuel consumption and CO2 emission is presented, due to their relevance to our problem.

## 2.1 Robustness in Airline Planning Process

In order to overcome the negative effects of the unexpected disruptions in the airline processes, at the planning stage robustness is considered. The robustness in airline processes aims to generate a plan which is less sensitive to disruptions [3], [5], [6]. Being less sensitive to disruptions can be achieved in different ways, so these different ways bring out various criteria and objectives for generating a robust plan. In the literature, the methods used to create a robust schedule are discussed by Lan, [7, p. 29], they are listed as follows:

- minimize some cost such as the cost for the worst case among all possible realizations of uncertainties.
- minimize aircraft/passenger/crew delays and/or disruptions
- maximize easiness of recovery aircraft/passenger/crew when disruptions occur
- isolate delays and schedule disruptions to avoid downstream impacts

Airline schedule planning problems mainly consider these four objectives and their combinations when robustness is addressed.

As another crucial study in the robustness in airline planning process, Arıkan et al. [8] present a stochastic model of delay propagation. The distinction between their study and the other studies in the literature is in the modeling of block times of downstream flight legs. They use a stochastic model of block times of all flight legs, whereas other studies such as Lan [7], Ahmadbeygi et al. [9] etc. assume deterministic block times given a random delay for the root flight. In this way, the impact of expected total propagated in the airline network is estimated more precise.

In the remaining sections, how robustness is handled by different problems is explained within the corresponding problem type and the study. A detailed literature review on robustness in airline problems is presented by Weide [10, p. 55].

## **2.2 Airline Schedule Planning Problems**

In this section, we present a detailed literature review for three airline schedule planning subproblems. Initially we start with schedule design problem and continue with fleet assignment problem. Eventually we present a literature review on maintenance routing problem.

### **2.2.1 Schedule Design Problem**

Airline schedule design problem decides where to fly and in which frequency in consideration of market demand, profitability, available resources and the competitors [3]. Due to its broad scope, Barnhart, Belobaba and Odoni [11] state that building flight schedules from scratch is performed manually with limited optimization in the typical airline practice. In the recent literature generally a schedule augmentation problem is solved instead of constructing a schedule from

scratch [10], [11]. This augmentation considers flight cancellation, addition, departure time changes, idle time insertion in order to grasp market demand and profitability as well as achieve robustness.

Lan et al. [7] consider a flight re-timing model. In their study flight departure times can be changed in a time interval in order to achieve robustness in terms of minimizing passenger delay and disruption. They discretize departure times and create arcs for each possible departure time and solve the mixed integer programming problem by column generation and branch and bound method. In fact, re-timing flight departures implies changing slack (idle time) between flights. A similar study is conducted by Ahmadbeygi et al. [9]. They redistribute existing slack in the planning process, making minor modifications in departure times, however they try to minimize expected value of delay propagation in order to achieve robustness in terms of minimizing delays and avoiding downstream impacts. They propose two models which are single-layer model and multi-layer model. Single-layer model just considers the delay propagation at the flights directly connected to the flight causing delay while multi-layer model considers the delay propagation of a flight over all the downstream flights. In addition to changes on departure times of flights and idle time insertion; Duran et al. [12] propose a robust airline scheduling model which controls cruise time and satisfy passengers' connection service levels by chance constraints. In their study, the trade off between the costs of cruise time change and idle time insertion is considered while passengers' connection service levels are ensured by chance constraints. They propose a second order cone programming model. Especially for building flight schedules from scratch, Etschmaier and Mathaisel [13] present a literature review on airline scheduling.

### **2.2.2 Fleet Assignment Problem**

Fleet assignment problem tries to find the optimal assignment of aircraft types to flights by considering number of aircrafts in each fleet and coverage of all flights [3].

Two pioneering studies of basic fleet assignment problem are the study of

Abara [14] in which a connection network to model the flight network is used and the study of Hane et al. [15] in which the model is based on a time-line network. In addition to basic fleet assignment problem, an enhanced fleet assignment problems which consider network effects are presented. Jacobs et al. [16] propose a model which considers network effect and stochastic nature of demand. They use Benders decomposition to integrate the FAM model with the O&D revenue management model. Similarly Barnhart et al. [17] consider fleet assignment model from the point of network considerations in order to minimize net revenue lost due to spilled passengers; additionally they consider the option of recapturing spilled passengers from itineraries. They solve their model using a branch-and-price-and-cut algorithm in which columns and constraints are generated. A study which incorporates robustness into fleet assignment problem is conducted by Smith et al. [18]. In their study, a term named station purity which refers to the number of fleet types serving a station is introduced. Due to a better station purity, it is easier to recover aircraft/passenger/crew when disruption occurs. Moreover it is reported station purity concept provides benefits in terms of planned crew and maintenance costs. A detailed literature review for fleet assignment problem is presented by Sherali et al. [19].

### **2.2.3 Maintenance Routing**

After fleet assignment decomposes flight networks into subnetworks in terms of a particular fleet type, maintenance routing problem assigns individual aircrafts to these flights in consideration of the maintenance requirements [3]. The Federal Aviation Administration (FAA) requires several types of aircraft maintenance such as A check every 65 flight hours [20]. However each airline company has its own maintenance policy so it is possible to see that different models adopt different approaches for maintenance routing such that they do not violate regulations. These differences in policies cause different assumptions in models. For example, Clarke et al. [21] adopt a maintenance policy such that each aircraft enters maintenance every three days while Lapp [6] assumes maintenance once in a week. The common trait in these approaches is that they consider the maintenance check which is the most frequent since other checks require a long planning

horizon. In Table 2.1 as cited in [22], maintenance check types, their frequencies and the duration of these checks are illustrated.

Table 2.1: Maintenance checks

Type A	65 FH	One Night
Type B	300-600 FH	One Night
Type C	1 Year	One Month
Type D	4 Year	One Month

Clarke et al. [21] propose a study which models the maintenance routing problem as an asymmetric traveling salesman problem with side constraints and solve the model using Lagrangian relaxation and subgradient optimization. They try to maximize the benefit derived from the making specific connections by considering through value issue which depends on marketing advantage of connections and ground time between flights. Gopalan and Talluri [20] introduce lines-of-flight concept. A lines of flight corresponds to a flight sequence which can be operated during a day. After constructing the set of lines-of-flight, they generate routes sustaining that an aircraft visits a maintenance station once every three days or less and at least once through the balance-check station. Lapp [6] also adopts lines-of-flight concept in his dissertation, additionally maintenance lines of flight concept is introduced which refers to a lines of flight ending at a maintenance station. Lapp incorporate robustness into maintenance routing problem by minimizing the total number of expected maintenance misalignments. Maintenance misalignments of a station refers to the difference between the number of maintenance requiring aircraft which starts the day at that station and the number of maintenance lines of flights originating at that station. Haouari et al. [23] propose a model for daily maintenance routing problem in which they ensure maintenance feasibility by counter constraints on flight hours, take offs and number of days since the last maintenance checks for each aircraft. They present a compact polynomial-sized representation for the general aircraft routing model and they linearize and lift that representation. Moreover, in the study of Aloulou et al. [24], a MIP model is proposed for the robust aircraft routing problem without directly accommodating maintenance constraints however by considering that the flights start and end in the single hub where maintenance

checks are achieved overnight. Aloulou et al. [24] capture robustness by an objective function pertaining to aircraft and passenger connections. Literature review for aircraft routing problem can be found in the study of Gopalan and Talluri [4].

## 2.3 Integrated Problems

In this section, we present a literature review for integrations of airline schedule planning subproblems which are elaborated in the previous section. The integrated problems which are considered are schedule design and fleet assignment problem, fleet assignment and aircraft routing problem and lastly schedule design, fleet assignment and aircraft routing problem.

### 2.3.1 Schedule Design and Fleet Assignment

Integration of schedule design and fleet assignment decides simultaneously on fleet assignment and schedule design in terms of adding/canceling flights or changing departure times.

Rexing et al. [25] propose a model which considers departure re-timing and fleet assignment simultaneously and they show that this integration, schedule design problems consider the fleet capacities, so it is possible to improve fleet decision in terms of spill cost and aircraft productivity. They discretize each possible departure time for each flight and solve the model in two different algorithmic ways: direct solution approach which is good for speed and simplicity and iterative solution approach which is good for memory usage. Lohatepanont and Barnhart [26] consider schedule design and fleet assignment in an integrated way in which a base schedule and two flight lists including mandatory and optional flights are given. Starting from the base schedule they consider deleting/adding flights from/to the base schedule with respect to given flight lists. In order to solve their model, they use an iterative algorithm in which column generation is used and demand correction terms are revised at each iteration. In a similar fashion, Sherali et al. [27] propose a model that integrates the schedule design and fleet assignment processes while considering flexible flight times, schedule balance,

and recapture issues, along with optional legs, path/itinerary-based demands, and multiple fare-classes. Differently, they consider the flow of passengers along itineraries over the network together with flight scheduling and fleet assignment decisions in order to maximize profits while Lohatepanont and Barnhart [26] makes their main model feed leg selections and fleet assignment decisions along with itinerary demands into a passenger mix model which is solved subsequently. They generate valid inequalities to tighten their model and then apply Benders decomposition method to the resulting tightened model. Şafak [28] integrates fleet assignment problem and the robust airline scheduling model suggested by Duran et al. [12]. While making fleet assignment and modifying a given schedule in terms of departure time, cruise time control or idle time insertion in order to achieve robustness, they consider the differences of each fleet type such as fuel efficiency, seat capacity, CO2 emission capacity and idle time cost. They propose a second order cone programming model and a two phase solution algorithm for large instances which give near optimal solutions.

### **2.3.2 Fleet Assignment and Aircraft Routing**

This integrated problem aims to cover all flights by an aircraft while constructing maintenance feasible routes for each aircraft simultaneously. Separate fleet assignment problem does not consider maintenance feasibility and route construction while assigning fleet types to flights, therefore, the output of fleet assignment may yield a solution which is infeasible in terms of maintenance [3]. However in an integrated problem of fleet assignment and aircraft routing, it is guaranteed that maintenance constraints for each aircraft is preserved.

Barnhart [29] et al. integrate fleet assignment and aircraft routing by defining flight strings. Flight strings start and end at a maintenance station so they are maintenance feasible. Although this approach cause millions of strings, they solve their model with branch and price solution method. As distinct from string approach, Grönkvist [30] suggests a multi-commodity network flow model with side constraints for integrated fleet assignment and aircraft routing and defines this problem as tail assignment problem. In that study, maintenance requirements



are controlled by counter constraints for each maintenance parameter. The model is solved by a method which uses column generation and constraint programming. As a more recent study, Liang et al. [31] propose an integrated model for a weekly planning horizon and introduce weekly rotation tour network model. In order to integrate two problems they create weekly rotation tour network for each fleet type and solve the model by using a diving heuristic method efficiently. Briefly diving heuristic method, an iterative heuristic to fix the variables based on their values in the LP solution.

### **2.3.3 Schedule Design, Fleet Assignment and Aircraft Routing**

Integrating three problems enables to improve local optimal solutions which are found by solving separately, however tractability worsens as much as the scope of integration expands. Therefore this integration problems are generally modeled and solved for daily planning horizon.

Desaulniers et al. [32] integrate three problems within a daily planning horizon and suggest two different formulations for the same problem. First is a set partitioning and the second is a multi-commodity network flow problem. Both problems are solved by column generation and branch-and-bound method. As a more recent study, Sherali et al [33] propose an approach in which they integrate the schedule design, fleet assignment, and aircraft-routing problems within the consideration of flight selection, departure timing and maintenance requirements. For maintenance requirements, they use a limit on total flight time of each aircraft. The total flight time of a daily route for an aircraft is less than the limit and also the remaining flight hours from that limit at the end of the day are sufficient to ferry that aircraft from the last airport of the day to the nearest maintenance station. They also use a multiplier which changes flight hour limit for each fleet type. As a solution method, they use Benders' decomposition and enhance the model via valid inequalities. Eventually, it is worthwhile to mention that Papadakos [34] proposes an approach which integrates crew assignment problem to these three stages and solves by using Benders' decomposition.

## 2.4 Second Order Cone Programming

Second order cone programming has gained a significant place in recent years due to its capability of handling non-linear problems. Moreover, the reformulations techniques for 0,1-mixed integer nonlinear programs proposed by Günlük and Linderoth [2] provide modeling flexibility for the problems in which indicator variables open/close some constraints. They express the convex hull via conic quadratic constraints, so relaxations can be solved via second-order cone programming. There are various implementations of conic reformulations in different studies. For example, Aktürk et al. [35] studied conic quadratic reformulations to solve machine job assignment problem with separable convex cost functions. In addition, the examples of conic quadratic reformulations in airline scheduling problems can be seen in the studies by Şafak [28] and Duran et al. [12].

Second order cone programming have a crucial place in our study. The non-linear cost function is handled by second order conic equations. Hence, our non-linear problem is transformed into a solvable problem in commercial solvers. More information about conic programming and conic representable functions can be found in Ben-Tal and Nemirovski [1].

## 2.5 Cruise Speed Control versus Fuel Consumption and CO2 Emission

According to IATA's [36] analysis on airline financial data, while the share of the fuel cost was 12-13% between 2001 and 2003; it was 32.3% of the total airline cost in 2008. Due to high fuel price and the additional cost of CO2 emission caused by fuel consumption, fuel has been the largest single cost term for the global airlines.

Although fuel cost is the largest cost term, choosing cruise speed which minimizes fuel cost might cause higher time related cost such as maintenance, crew and ownership or rental cost. Minimum fuel cost requires lower cruise speed than the time related costs require. Due to this trade off, Airbus [37] presented a cost index function to balance these cost factors and help to select the best speed

while minimizing the overall cost for each flight.

However, due to the network effect of each flight on passenger and aircraft connections, the decision of cruise speed versus fuel cost shouldn't be locally made. A cruise speed decision which minimizes the cost of a flight might deteriorate the cost of other connecting flights. Hence various studies consider the trade off between cruise speed control and fuel cost within the consideration of network effect and other operational costs. Aktürk et al. [38] propose a recovery model which is using controllable cruise speed first time in a recovery model. Arıkan et al. [39] also propose a recovery model for passenger and aircraft recovery problem by integrating cruise speed control along with retiming of the departure times of flights and swapping aircraft. Moreover, for robust airline scheduling, Duran et al. [12] present a mathematical model in which the trade off between cruise speed control and idle time insertion. Eventually, Şafak [28] integrates robust airline scheduling and fleet assignment problem within the consideration of cruise speed control.

A detailed literature review on cruise speed control versus fuel consumption and CO2 emission is presented by Şafak [28].

## 2.6 Summary

In this chapter, we present a literature review related to airline schedule design problems. First, we start with major problems and then continue with the integration of them. Meanwhile, we emphasize the link to robustness in each problem. Furthermore, we address the necessary literature for our problem definition such as second order cone programming and cruise speed control versus fuel consumption and CO2 emission.

# Chapter 3

## Problem Definition

Our problem is to solve the robust airline schedule design, aircraft fleetings and routing problems within a daily planning horizon for a given set of flights and a set of aircraft in an integrated manner while considering maintenance requirements and passengers' connection service levels.

The given information regarding a flight is departure time window, cruise time window, origin airport, destination airport, expected demand, the opportunity cost incurred when a passenger is spilled and the distribution of non-cruise time for that flight. Moreover minimum required turn around time between two flights is also known depending on each aircraft as well as minimum required times for passengers' connections among flights. Eventually congestion coefficient for each airport is also known and the distribution of non-cruise time of a flight depends on the congestion coefficients of origin and destination airports.

The given information regarding an aircraft is seat capacity, fuel consumption and CO2 emission cost parameters, idle time cost, limit on the total cruise time and the airport on which it has to land to ensure maintenance requirements and the first airport from which it could fly; lastly the daily usage cost incurred when that aircraft is used. Daily usage cost refers to the sum of the fixed operating costs and the opportunity cost which can be thought as the value of using that aircraft as buffer in case of a disruption in the usual schedule.

The robust airline scheduling, fleet and routing model determines for each flight: idle time insertion, cruise time change, departure time and an individual aircraft to use. For each aircraft, the model determines whether that aircraft is going to be used and, if it is used, the flight sequence to be flown for that particular aircraft. These decisions are made in consideration of the idle time insertion cost, fuel consumption and CO2 emission costs, spill cost and aircraft usage cost.

The feasible set of the model satisfies maintenance requirements, passengers' connection service levels and flight connection constraints in cases two flights are performed by the same aircraft consecutively. For maintenance requirements, we adopt two basic rules for each aircraft. The first one is that we limit the total cruise time for each aircraft on a day. A similar approach is used by Sherali et al. [33], they use a limit ( $\lambda_t$ ) on total flight hours of each aircraft on a day. In our problem, we take the flight time as the sum of cruise and non-cruise time. The non-cruise time is a known parameter for each flight so we choose to use a limit on cruise time which is a decision variable in our problem. Therefore, we prefer that limit to be smaller than the flight hour limit as much as the total possible non-cruise time of an aircraft on a day. The second one is that the first and the last airport of an aircraft is predetermined on a day. In case when an aircraft is used, in order for that aircraft to follow its ordinary maintenance checks, we secure that aircraft takes off from/lands to the particular airports at the beginning/end of a day. For passengers' connection, we use service levels with chance constraints and ensure necessary time for passengers to make connection with a particular probability, service level, between each pair of flights which is feasible for passengers' connection. Eventually, we provide minimum turn around time between flights which are performed by the same aircraft.

### 3.1 Distribution of Non-cruise Times

Similar to the studies of Duran et al. [12], Şafak [28], in our model we take the flight time as the sum of cruise time and non-cruise time. Non-cruise time consists of the taxi-in and taxi-out stages as well as climb and descend stages of a flight

which include uncertainty depending on the airport congestion or weather conditions while cruise time refers to the time which is controllable by speeding up the aircraft. Hence in our study, we take cruise time as a decision variable and non-cruise time as a random variable.

Arikan and Deshpande [8] show that the log-Laplace distribution provides a good fit to the block time of a flight. Therefore, for each flight  $i \in F$ , random variable  $NC_i$ , which represents the non-cruise time of flights is assumed to be log-Laplace distribution with two parameters,  $\alpha$  and  $\beta$ . For each flight  $i \in F$ ,  $\beta_i$ 's are calculated by multiplying the parameter  $\beta$  with a function,  $g$  of origin and destination airports' congestion factors. It is given as:

$$\beta_i = \beta \cdot g(e_{Or_i}, e_{Dn_i}) \quad (3.1)$$

where  $Or_i$  and  $Dn_i$  are the origin and destination airports of flight  $i \in F$  respectively. Therefore, the mean and variance of the random variable depend on the congestion factors of the origin and destination airports. It means that, if a flight arrives or departs from a congested airport, the probability of non-cruise stage of that flight requires more time is higher. To guarantee passengers' connection service level, we establish chance constraints over the random variable  $NC_i$  of non-cruise time.

### 3.1.1 Log-Laplace Distribution

The probability density function and cumulative distribution function of Log-Laplace random variable  $X$  with a scale parameter,  $e^\alpha$  and the tail parameter,  $1/\beta_i$  is given as:

$$f_X(x) = \begin{cases} \frac{1}{2 \cdot \beta_i \cdot x} e^{\frac{(\ln(x) - \alpha)}{\beta_i}} & \text{if } \ln(x) < \alpha \\ \frac{1}{2 \cdot \beta_i \cdot x} e^{-\frac{(\ln(x) - \alpha)}{\beta_i}} & \text{if } \ln(x) \geq \alpha \end{cases}$$

$$F_X(x) = \begin{cases} \frac{1}{2} e^{\frac{(\ln(x) - \alpha)}{\beta_i}} & \text{if } \ln(x) < \alpha \\ 1 - \frac{1}{2} e^{-\frac{(\ln(x) - \alpha)}{\beta_i}} & \text{if } \ln(x) \geq \alpha \end{cases}$$

The quantile function of the log-Laplace distribution is given as:

$$F_X^{-1}(p) = \begin{cases} (2p)^{\beta_i} \cdot e^\alpha & \text{if } p < 1/2 \\ \frac{e^\alpha}{(2-2p)^{\beta_i}} & \text{if } p \geq 1/2 \end{cases}$$

In the study of Duran et al. [12], it is shown that mean is finite if and only if  $\beta_i < 1$  and it is given as:

$$\mathbb{E}[X] = \frac{e^\alpha}{(1 - \beta_i) \cdot (1 + \beta_i)} \quad (3.2)$$

## 3.2 Fuel Consumption and CO2 Emission Cost

The parameters which are required to calculate fuel consumption are aircraft properties and the coefficients related to physical conditions.  $C_{f1}, C_{f2}, C_{D0,CR}, C_{D2,CR}, C_{fcr}$ , mass ( $m$ ) and surface ( $S$ ) are known parameters for an aircraft and they are available in EUROCONTROL [40].  $d$  refers to the distance flown at the cruise stage. Air density  $\rho$  and gravitational acceleration  $g_0$  are also known as well as bank angle  $\phi$ .

In the study of Şafak [28], with these available parameters, it is shown that the following equation gives the total fuel consumption in kg.

$$F_i^t(f_i^t) = c_1^{i,t} \cdot \frac{1}{f_i^t} + c_2^{i,t} \cdot \frac{1}{(f_i^t)^2} + c_3^{i,t} \cdot (f_i^t)^3 + c_4^{i,t} \cdot (f_i^t)^2 \quad (3.3)$$

where,

$$c_1^{i,t} = \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot C_{D0,CR}^t \cdot \rho \cdot S^t \cdot d_i^2 \quad (3.4)$$

$$c_2^{i,t} = \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot \frac{C_{D0,CR}^t \cdot \rho \cdot S^t \cdot d_i^3}{C_{f2}^t} \quad (3.5)$$

$$c_3^{i,t} = \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot \frac{C_{D2,CR}^t \cdot 4 \cdot m_t^2 \cdot g_0^2}{\rho \cdot S^t \cdot \cos(\phi)^2 \cdot d_i^2} \quad (3.6)$$

$$c_4^{i,t} = \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot \frac{C_{D2,CR}^t \cdot 4 \cdot m_t^2 \cdot g_0^2}{C_{f2}^t \cdot \rho \cdot S^t \cdot \cos(\phi)^2 \cdot d_i^2} \quad (3.7)$$

When fuel consumption is known, the fuel consumption and CO2 emission cost is calculated by multiplying that amount with the cost coefficients ( $c_{fuel} + c_{CO_2}$ ).

### 3.3 Service Level

In this study, similar to the studies of Duran et al. [12] and Şafak [28], passengers' connections are taken into account to develop a robust schedule such that misconnections of passengers are minimized when a disruption occurs. Between two flights  $i$  and  $j$ , if the origin airport of the flight  $j$  is the same as the destination airport of the flight  $i$  and the departure time of flight  $j$  is later than the arrival time of the flight  $i$ , the time needed for the passengers' connection is  $TP_{ij}$ . The percentage of the passengers' connection satisfied between flights  $(i, j)$  is represented by the decision variable  $\gamma_{ij}$ . However, while in those studies service level is taken as an objective to be maximized, in our study, we adopt predetermined values for service level and add the chance constraints for those predetermined values.

### 3.4 Numerical Example

In order to elaborate our problem definition and the model mechanics, in this section, we provide a numerical example. First, we present a published schedule which shows planned departure time, flight duration, idle time and turn around time. Table 3.1 shows the published schedule and in Figure 3.1 we illustrate the time space network of that schedule. In our approach, a new schedule and aircraft fleetings and routing are generated within the consideration of idle time cost, fuel consumption and CO2 emission costs, spill cost and daily usage cost. While generating the proposed schedule, the model considers changes up to 15 min. on departure times, idle time insertion, cruise time change as well as aircraft assignment and routing changes. The proposed schedule is feasible in terms of maintenance requirements since all aircrafts land to the same airports as in the published schedule and in the meantime we limit the flight hours of each aircraft regarding maintenance requirements. Meanwhile passengers' service levels are



ensured by the chance constraints in the proposed schedule, for this numerical example minimum 95% service level is adopted. The proposed schedule is shown in Table 3.3 and in Figure 3.2, the time space network can be seen.

Table 3.1: Published schedule

Tail#	Flight #	From	To	Dep. Time	Duration	Cruise Time	Idle Time	TA
N3ELAA	2057	ORD	SJU	08:30	290	270	35.6	29.4
	2078	SJU	ORD	14:25	335	315	-	-
N3DUAA	2099	ORD	LAX	07:00	270	250	34.9	35.1
	1972	LAX	ORD	12:40	245	225	24.38	35.6
	1972	ORD	RDU	17:45	115	95	-	-
N412AA	2345	ORD	DFW	17:15	155	135	2.5	47.5
	2374	DFW	ORD	20:40	130	110	-	-
N4XGAA	2079	ORD	SAN	08:45	270	250	13.5	31.5
	1438	SAN	ORD	14:00	250	230	58.9	41.1
	346	ORD	LGA	19:50	135	115	-	-

The published schedule in Table 3.1 has 10 flights which are operated by 4 aircraft. For each flight, the information of departure time, duration which is sum of cruise and non-cruise time, turn around time, idle time and cruise time are given as well as origin and destination airports. To explain, the tail number N3ELAA performs flight 2057 from ORD to SJU at 08:30 in 290 minutes and before the next flight, flight 2078, N3ELAA spends 29.4 minutes for turn around time and stands idle for 35.6 minutes. In this study, it is assumed that, 20 minutes of the block times are given as the non-cruise time of the flights and the remaining times are given as the cruise time of the flights. For example, the duration column represents the block times of the flights and there are 20 minutes difference between cruise time and duration columns for each flight.

Figure 3.1 illustrates the time space network of the published schedule. The horizontal axis represents the time while the vertical axis represents the airports. For each flight, there is an angular arrow and the flight number nearby. The horizontal arrows represent the turn around and idle time of an aircraft in the corresponding airport. The route of each aircraft are drawn by a different line style. For example, the solid arrows represent the operations of tail number N4XGAA.

The cost calculation of the published schedule is shown in Table 3.2. The fuel consumption and CO2 emission costs calculations are explained in previous sections. Idle time costs are calculated by the multiplication of unit idle time cost



Table 3.2: Cost calculation for the published Schedule

Tail#	Flight #	Fuel & CO2	Idle Time	Spill	Daily Usage
N3ELAA	2057	7611	5334	0	90000
	2078	8879.5	-	0	
N3DUAA	2099	5923.4	4955.8	0	85200
	1972	5331	3461.9	0	
	1972	2250.9	-	0	
N412AA	2345	2319	347.2	0	84000
	2374	1889.6	-	0	
N4XGAA	2079	6509.4	1944	0	86400
	1438	5988.7	8481.6	0	
	346	2994.3	-	0	
Total		49696.8	24524.5	0	345600

Table 3.3: Proposed schedule

Tail#	Flight #	From	To	Dep. Time	Duration	Cruise Time	Idle Time	TA
N3DUAA	2079	ORD	SAN	08:38	270.4	249.5	8.4	27.3
	1438	SAN	ORD	13:45	219.4	198.4	0	35.6
	1972	ORD	RDU	18:00	115.5	94.8	0	-
N412AA	2099	ORD	LAX	06:55	280.6	249.9	0	48.6
	1972	LAX	ORD	12:25	252.6	221.7	0	49.3
	2345	ORD	DFW	17:26	160.4	133	0	47.5
	2374	DFW	ORD	20:55	137.8	110	0	-
N4XGAA	2057	ORD	SJU	08:45	290.3	275.9	1.2	27.6
	2078	SJU	ORD	14:10	313.9	293.6	0	41.1
	346	ORD	LGA	20:05	141.7	115	0	-

$$\text{Cost Improvement} = 100 \times \frac{\text{Published Schedule} - \text{Proposed Schedule}}{\text{Published Schedule}}$$

As mentioned before, total number of aircraft used is decreased from 4 to 3. In the proposed model, the most expensive aircraft in terms of daily usage cost is tail number N3ELAA and also N3ELAA is not a fuel efficient aircraft in comparison to N412AA and N3DUAA. In Figure 3.3, in order to illustrate fuel efficiency of the aircraft, the fuel emission and CO2 emission costs realizations are shown when all four aircraft perform the same flight, flight 1438. Due to its high daily usage cost and low fuel efficiency our model generates a schedule in which N3ELAA is not used and the other aircraft are utilized more. Although the fuel efficiency of aircraft N4XGAA is worse than N3ELAA, the model prefers

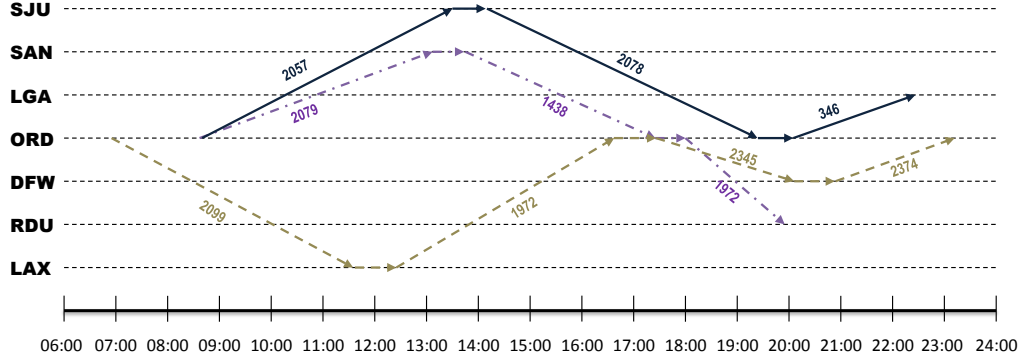


Figure 3.2: Time space network of the proposed schedule

to use N4XGAA since its daily usage cost is smaller than N3ELAA. As a direct consequence of better aircraft utilization, this change provides 26% improvement in the cost of daily usage.

Another effect of the utilization of efficient aircraft is on the fuel consumption and CO2 emission costs. Since utilizations of efficient aircraft are increased and better flight aircraft assignment is achieved, total cost due to fuel consumption and CO2 emission is improved around 10%. This improvement is achieved although cruise times of flights 2078, 1438 and 2345 are changed by 7%, 14% and 2% respectively. As an example of better flight aircraft assignment, in the published schedule flight 1438 is performed by aircraft N4XGAA with \$5988.7 fuel & CO2 cost, while in the proposed schedule same flight is performed by aircraft N3DUAA with \$5650 fuel & CO2 cost though it is compressed 14%. Since N3DUAA is more efficient than N4XGAA, the extra fuel & CO2 cost caused by that compression which is necessary to assign 1438 to N3DUAA is compensated and even smaller fuel & CO2 cost is achieved. On the other hand, as an example of increased utilization of efficient aircraft, in Figure 3.4 in which the total cruise time of each aircraft in the published schedule by the first column and the proposed schedule by the second column, it is seen that the utilization of N412AA is increased most. Besides, in Figure 3.3, it can be seen that N412AA is the most fuel efficient aircraft.

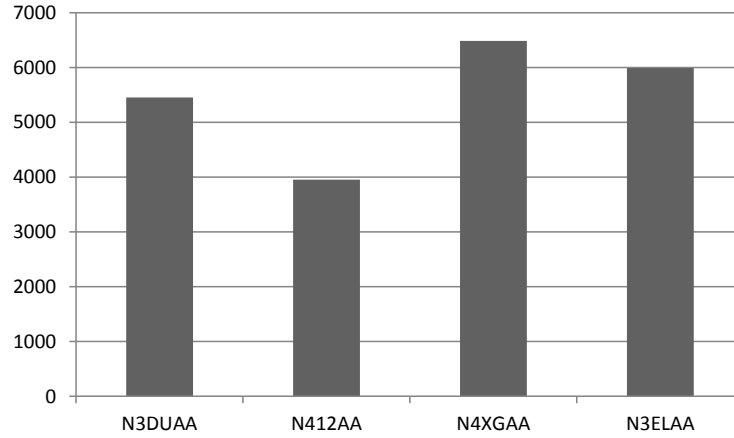


Figure 3.3: The fuel & CO2 cost of flight 1438 with each aircraft

The effect of routing, cruise time and departure time control improve the cost of the idle time insertion as well as the consideration of non-cruise time distribution. The necessity of idle time insertion after flights can be eliminated by changing routing decisions, cruise time and departure time. Moreover while in the published schedule, the non-cruise time is taken as 20 minutes regardless of the randomness on the non-cruise time, in the proposed model non-cruise time distribution of each flight is considered as mentioned in Section 3.1. The improvement on the idle time cost is 94%.

While there is an improvement in sum of daily usage cost, idle time cost, fuel consumption and CO2 emission costs, the spill cost is increased in the proposed schedule. The reason of this increase is that the fleet assignment in the published schedule is generated in consideration of aircraft capacity and passenger demand, however, in the proposed model the other cost terms are also considered. The improvement in the other cost terms overcomes the increase in the spill cost. While in the published schedule the spill cost is 0, the proposed model has a spill cost \$2022.1. The spill of passengers are occurred in flights 2079, 1438, 2099 and 1972 in the proposed schedule.

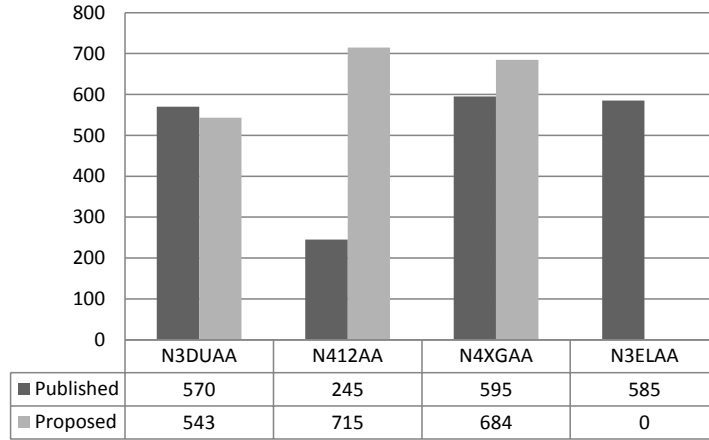


Figure 3.4: The total cruise time in the published and the proposed schedules

When we compare the overall results, the total cost is improved 27% in the proposed schedule. The costs of both schedules are calculated at the planning stage. After the realizations of non-cruise times of each flight, the actual cost of the idle time and delay costs can be seen. For this purpose a simulation study can be done to compare the realized costs of the schedules. However, with this numerical example, we aim to show how the mechanics of the proposed model work.

### 3.5 Summary

In this chapter, we give a definition for our problem while explaining the input and the decisions. Meanwhile, the necessary concepts in this definition such as service level, distribution of non-cruise time and fuel consumption and CO2 emission cost, maintenance requirements are discussed. Moreover, to elaborate we present a numerical example over a small data which illustrates the trade offs our of our problem.

Table 3.4: Cost calculation for proposed schedule

Tail#	Flight #	Fuel & CO2	Idle Time	Spill	Daily Usage
N3DUAA	2079	5923.3	1192.8	453.1	85200
	1438	5650	0	431.5	
	1972	2250.8	0	0	
N412AA	2099	4294.5	0	443.9	84000
	1972	3866.4	0	693.6	
	2345	2320.4	0	0	
	2374	1889.6	0	0	
N4XGAA	2057	7035.2	172.8	0	86400
	2078	8260.5	0	0	
	346	2994.3	0	0	
	Total	44485	1365.6	2022,1	255600

# Chapter 4

## Problem Formulation

There is a profound correlation among the cost terms and this yields various trade offs for our problem. For example, the flight networks with small number of aircraft are favorable for daily usage cost and idle time cost while they are unfavorable for spill cost and fuel consumption and CO2 emission cost. As another example, while an aircraft causes spill cost for a flight, it might decrease the fuel consumption and CO2 emission cost due to its efficiency. On the other hand, in order to ensure passengers' connection service level, compressing cruise time might be favorable instead of inserting idle time. Moreover, there are certain conditions which have to be satisfied by any generated schedule such as passengers' connection service levels and maintenance requirements.

Initially we present the notation which is used in the mathematical formulation below.

### Sets

$T$	: Set of aircrafts which can be used
$F$	: Set of flights which have to be performed by an aircraft
$U^i$	: Set of flights which can connect to flight $i$
$D^i$	: Set of flights which flight $i$ can connect
$B$	: Set of airports
$F_e^t$	: Set of flights which aircraft $t$ can use as a last flight in the schedule



- $F_s^t$  : Set of flights which aircraft  $t$  can use as a first flight in the schedule  
 $P_i$  : Set of flights that have a passenger connection with flight  $i$   
 $A$  : Set of flights  $i$  and  $j$  such that flight  $i$  can connect flight  $j$

### Parameters

- $Idle_t$  : Unit cost of the idle time of aircraft  $t$   
 $Cap_t$  : Seat capacity of aircraft  $t$   
 $Dem_i$  : Passenger demand of flight  $i$   
 $Daily_t$  : The cost incurred when aircraft  $t$  is used  
 $TA_{ij}^t$  : Turntime needed to prepare aircraft  $t$  between flights  $i$  and  $j$   
 $Cspl_i$  : Opportunity cost of spilled passengers of flight  $i$   
 $\lambda_t$  : The total available cruise time of aircraft  $t$  on a day  
 $\gamma_{ij}$  : Passengers' connection service level between flights  $i$  and  $j$   
 $c_{fuel}$  : Cost of fuel per kg of aircraft fuel consumption  
 $c_{CO2}$  : Cost of emission per kg of aircraft CO2 emission  
 $f_i^u, f_i^l$  : Upper and lower limit of the cruise time of flight  $i$   
 $d_i^u, d_i^l$  : Upper and lower limit of the departure time of flight  $i$   
 $e_b$  : Airport congestion coefficient for airport  $b$   
 $Or_i$  : Origin airport of flight  $i$   
 $Dn_i$  : Destination airport of flight  $i$   
 $NC_i$  : The random parameter denoting the non cruise of flight  $i$

### Decision Variables

- $x_{ij}^t$  : 1 if flight  $i$  is followed by flight  $j$  performed by aircraft  $t$  and is 0 ow.  
 $y_i^t$  : 1 if flight  $i$  is the first flight performed by aircraft  $t$  and is 0 ow.  
 $z_i^t$  : 1 if flight  $i$  is the last flight performed by aircraft  $t$  and is 0 ow.  
 $d_i$  : Departure time of flight  $i$   
 $s_i^t$  : Idle time of aircraft  $t$  after flight  $i$   
 $f_i^t$  : Cruise time at flight  $i$  performed by aircraft  $t$

In the model,  $T$  represents the set of aircrafts which can be operated.  $F$  is the set of flight legs which have to be covered by using the aircraft in  $T$ . For each flight  $i$  in  $F$ , there are two sets which are  $U^i$ , upstream flights and  $D^i$ , downstream flights.  $U^i$  denotes the set of flights which can follow flight  $i$  in terms of departure time and origin, destination airports pair. For example, ORD-DFW

Dep: 09.00 exists in  $U^i$  where  $i$  is DFW-DCA Dep: 13.00. It is easy to check origin destination pair, the destination airport of a flight in  $U^i$  has to be same with the origin airport of  $i$ . For departure time, there has to be sufficient time for connection between the lower bound of departure time of a flight in  $U^i$  and the upper bound of the departure time of flight  $i$ ; therefore it can be stated that a flight in  $U^i$  and flight  $i$  might be performed by the same aircraft consecutively. If flight  $j$  is in  $U^i$  then, flight  $i$  is in  $D^j$ .

$B$  denotes the set of airports to be considered in the model.  $F_e^t$  denotes the set of flights for an aircraft  $t$  in  $T$  such that the flights in  $F_e^t$  can be the last flight of aircraft  $t$  since they terminate at the convenient airports in terms of maintenance feasibility.  $F_s^t$  denotes the set of flights for an aircraft  $t$  in  $T$  such that the origin airport of flights in  $F_s^t$  are the airport from which that aircraft starts the planning day.

$A$  denotes the set of flights  $(i, j)$  such that flight  $j \in D^i$ , i.e. flight  $i \in U^j$ . If  $(i, j)$  is in  $A$ , it means that an aircraft can fly these flights consecutively.  $P_i$  is the set of flights such that the passengers in flight  $i$  might have connection.

Binary decision variable  $x_{ij}^t$  takes value 1 in the model, if flight  $i$  and  $j$  are performed by the aircraft  $t$  consecutively for every  $(i, j)$  pair in  $A$  and  $t$  in  $T$  and 0 otherwise. Binary decision variable  $y_i^t$  takes value 1 in the model, if flight  $i$  is the first flight performed by the aircraft  $t$  and 0 otherwise; binary decision variable  $z_i^t$  takes value 1 if flight  $i$  is the last flight performed by the aircraft  $t$  and 0 otherwise.  $d_i$  is decision variable denoting the departure time of flight  $i$ . The decision variables  $s_i^t$  and  $f_i^t$  denote the idle time after flight  $i$  and cruise of flight  $i$  when flight  $i$  is performed by aircraft  $t$ .

## 4.1 Mathematical Model

In order to find the optimal solution to the integrated robust airline scheduling, aircraft fleet and routing with cruise speed control problem we developed a nonlinear mixed integer programming model. For each  $i \in F$  and  $t \in T$ , we redefine the fuel consumption:

$$F_i^t(f_i^t) = \begin{cases} \left( c_1^{i,t} \frac{1}{f_i^t} + c_2^{i,t} \frac{1}{(f_i^t)^2} + c_3^{i,t} (f_i^t)^3 + c_4^{i,t} (f_i^t)^2 \right) & \text{if } \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 1 \\ 0 & \text{if } \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 0 \end{cases}$$

so that if aircraft  $t$  is not assigned to flight  $i$ , then  $F_i^t(f_i^t) = 0$ .

The proposed nonlinear mathematical model is provided below:

$$\begin{aligned} \min \quad & \sum_{t \in T} \sum_{i \in F} \left( \sum_{j \in U^i} x_{ji}^t + y_i^t \right) \cdot C_{spl_i} \cdot \max(0, Dem_i - Cap_t) + \\ & \sum_{i \in F} \sum_{t \in T} (c_{fuel} + c_{CO_2}) \cdot F_i^t(f_i^t) + \end{aligned} \quad (4.1)$$

$$\begin{aligned} & \sum_{i \in F} \sum_{t \in T} y_i^t \cdot Daily_t + \sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t \\ \text{s.to} \quad & \sum_{j \in U^i} x_{ji}^t + y_i^t - \sum_{j \in D^i} x_{ij}^t - z_i^t = 0 \quad \forall i \in F, t \in T \end{aligned} \quad (4.2)$$

$$\sum_{i \in F} y_i^t \leq 1 \quad \forall t \in T \quad (4.3)$$

$$\sum_{t \in T} \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 1 \quad \forall i \in F \quad (4.4)$$

$$\begin{aligned} \text{IF} \quad & \sum_{t \in T} x_{ij}^t = 1 \\ \text{THEN,} \quad & d_j - d_i - TA_{ij} - \sum_{t \in T} f_i^t - E[NC_i] - \sum_{t \in T} s_i^t = 0 \quad \forall (i, j) \in A \end{aligned} \quad (4.5)$$

$$Pr \left[ NC_i \leq d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \right] \geq \gamma_{ij} \quad \forall i \in F, j \in P_i \quad (4.6)$$

$$\sum_{i \in F} f_i^t \leq \lambda_t \quad \forall t \in T \quad (4.7)$$

$$\begin{aligned} \text{IF} \quad & \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 1 \\ \text{THEN,} \quad & f_i^l \leq f_i^t \leq f_i^u \quad \forall i \in F, t \in T \end{aligned} \quad (4.8)$$

ELSE

$$f_i^t = 0$$

$$s_i^t = 0$$

$$y_i^t = 0 \quad \forall t \in T, i \in F \setminus F_s^t \quad (4.9)$$

$$z_i^t = 0 \quad \forall t \in T, i \in F \setminus F_e^t \quad (4.10)$$

$$d_i^l \leq d_i \leq d_i^u \quad \forall i \in F \quad (4.11)$$

$$s_i^t \geq 0 \quad \forall i \in F, t \in T \quad (4.12)$$

$$x_{ij}^t \in \{0, 1\} \quad \forall (i, j) \in A, t \in T \quad (4.13)$$

$$y_i^t \in \{0, 1\} \quad \forall i \in F, t \in T \quad (4.14)$$

The objective function, (4.1) is the sum of the spill cost, fuel consumption and

$CO_2$  emission cost, idle time cost and the daily aircraft usage cost. Constraint (4.2) is network balance equation. Constraint (4.3) ensures that each aircraft can be used for at most one flight sequence (string or path). Constraint (4.4) guarantees that each flight can be performed by exactly one aircraft. Constraint (4.5) ensures that if two flights are performed by the same aircraft consecutively, time between the departures of these flights have to be greater than the sum of cruise time, non-cruise time and turnaround time as much as idle time at the end of the first flight. Constraint (4.6) is the chance constraint which ensures the service level of passengers' connection. It guarantees that service level is greater than a determined percentage. Constraint (4.7) ensures that total cruise time of an aircraft does not exceed a predetermined time limit in order to ensure maintenance feasibility. If a flight  $i$  is performed by aircraft  $t$  then constraint (4.8) limits cruise time change; cruise time of a flight can not exceed the upper and lower bounds, else the corresponding variables  $f_i^t$  and  $s_i^t$  are set to zero. The aim of constraints (4.9) and (4.10) is to sustain maintenance policy intended in the published schedule, in this fashion first and last airport for each aircraft is determined. Constraint (4.11) sets the upper and lower bounds for departure time of each flight. Constraint (4.12) makes idle time stay non negative. Constraints (4.13) and (4.14) guarantees that  $x_{ij}^t$  and  $y_i^t$  are binary variables and due to (4.2) all  $z_i^t$  are also binary variables.

#### 4.1.1 Challenges for Solving the Model

The integrated robust airline scheduling, aircraft routing and fleet management is a hard problem in many aspects. The reasons which make the problem hard to solve can be listed as follows.

- Nonlinearity caused by the fuel consumption function
- Aircraft routing is an NP-complete problem [41].
- Disproportionate cost coefficients

In the objective function, the fuel consumption and CO2 emission cost functions are nonlinear functions and they also involve binary variables. Hence, this nonlinearity including binary variables is handled by the second order conic inequalities with binary variables. The details are presented in the following section.

Even if it is a special case of our problem, Parmentier [41] show that aircraft routing problem is an NP-complete problem. In addition to aircraft routing problem, we consider robust airline scheduling and fleet type assignment problems in an integrated fashion. Due to the integration, in the problem there are large number of decision variables and also aircraft routing problem is an NP-complete problem itself. For that reason, when the number of flights and aircrafts increases, the problem size increases drastically. Moreover, we also consider passengers' connection service levels with chance constraints as well as departure timing, idle time insertion and cruise speed control different from the aircraft routing problem.

In our study, the coefficients in the fuel consumption function are either very small or very large. This yields numerical stability problems with default parameter values of CPLEX solver. In order to avoid, this numerical stability problems, we change some parameter values and emphasize precision with consequent performance trade-offs in time and memory.

All of these reasons make our problem challenging in terms of both theoretic and numeric manners.

## 4.2 Reformulation of the Model

Reformulation of the model provides an exact solution for chance constraints and nonlinear objective functions as opposed to approximation methods. Using second order cone programming, the conic reformulation is achieved by representing the nonlinear objective term. Moreover, we express the chance constraints with closed forms.

### 4.2.1 Closed Form Expressions for the Chance Constraints

In Section 3.1.1., the quantile function of log-Laplace distribution is presented with parameters  $\alpha$  and  $\beta$  as follows:

$$F_X^{-1}(p) = \begin{cases} (2p)^{\beta_i} \cdot e^\alpha & \text{if } p < 1/2 \\ \frac{e^\alpha}{(2-2p)^{\beta_i}} & \text{if } p \geq 1/2 \end{cases}$$

The chance constraints in the model are in the form shown below.

$$Pr \left[ NC_i \leq d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \right] \geq \gamma_{ij} \quad \forall i \in F, j \in P_i$$

They can be expressed using quantile function of the probability distribution of random variable  $NC_i$ . The expression is as follows:

$$d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \geq F_X^{-1}(\gamma_{ij}) \quad \forall i \in F, j \in P_i$$

For a given value of  $\gamma_{ij}$ , the value of  $F_X^{-1}(\gamma_{ij})$  can be calculated.

### 4.2.2 Conic Representation of the Fuel Consumption and CO2 Emission Cost Functions

In the objective function, the cost functions involve nonlinearity due to controllable cruise time associated with the changing the cruise speed of the aircraft. To handle nonlinearity, nonlinear mixed integer optimization often requires too much computation time. On the other hand, it may not result in exact solutions. In order to shorten the solution time and obtain an optimal solution, in this section we show the conic quadratic reformulation of the fuel consumption function  $F_i^t(f_i^t)$  cost as discussed in Aktürk et al. [35] and Günlük and Linderoth [2]. To simplify the presentation, we drop the indices of the variables and parameters.

$$F(f) = \begin{cases} \left( c_1 \frac{1}{f} + c_2 \frac{1}{(f)^2} + c_3 (f)^3 + c_4 (f)^2 \right) & \text{if } w = 1 \\ 0 & \text{if } w = 0 \end{cases}$$

where  $w_i^t = \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right)$ , for simplicity.

$F(f)$  is discontinuous and therefore its epigraph  $E_F = \{(f, t) \in R^2 : F(f) \leq t\}$  is non-convex. In the next proposition, we describe how the convexity of  $E_F$  is obtained. A more detailed information can be found in Aktürk et al. [35] and Günlük and Linderoth [2].

**Proposition 1.** *The convex hull of  $E_F$  can be expressed as*

$$t \geq (c_1 \cdot q + c_2 \cdot \delta + c_3 \phi + c_4 v) \quad (4.15)$$

$$w^2 \leq q \cdot f \quad (4.16)$$

$$w^4 \leq f^2 \cdot \delta \cdot 1 \quad (4.17)$$

$$f^4 \leq w^2 \cdot \phi \cdot f \quad (4.18)$$

$$f^2 \leq v \cdot w \quad (4.19)$$

in the constraint set. Moreover, each of the inequalities (4.15)-(4.19) can be represented by conic inequalities.

*Proof.* Perspective of a convex function  $F(f)$  is  $wF(f/w)$  (Hiriart-Urruty and Lemaréchal [42]). Since each of the nonlinear terms  $\frac{1}{f}$ ,  $\frac{1}{f^2}$ ,  $f^3$  and  $f^2$  is a convex function for  $f \geq 0$ , then the epigraph of the perspective of each term can be stated as

$$\frac{w^2}{f} \leq q$$

$$\frac{w^4}{f^2} \leq \delta$$

$$\frac{f^3}{w^2} \leq \phi$$

$$\frac{f^2}{w} \leq v$$



respectively. Since  $w, f \geq 0$ , they can be written as stated in the proposition.

Finally, observe that (4.16) and (4.19) are hyperbolic inequalities, (4.17) can be restated as two hyperbolic inequalities

$$w^2 \leq u \cdot f \text{ and } u^2 \leq \delta \cdot 1$$

and (4.18) can be restated as

$$f^2 \leq u \cdot w \text{ and } u^2 \leq \phi \cdot f$$

which can be written as a conic quadratic inequality. □

Detailed transformations of each constraint are presented in the study of Şafak [28].

### 4.2.3 Conic Reformulation of the Model

With the closed form of the chance constraint and nonlinear cost terms as second order conic inequalities, the model becomes:

$$\begin{aligned}
\min \quad & \sum_{t \in T} \sum_{i \in F} \left( \sum_{j \in U^i} x_{ji}^t + y_i^t \right) \cdot Cspl_i \cdot \max(0, Dem_i - Cap_t) + \\
& \sum_{i \in F} \sum_{t \in T} (c_{fuel} + c_{CO_2}) \cdot (c_1^{i,t} q_i^t + c_2^{i,t} \delta_i^t + c_3^{i,t} \phi_i^t + c_4^{i,t} v_i^t) + \\
& \sum_{i \in F} \sum_{t \in T} y_i^t \cdot Daily_t + \sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t
\end{aligned} \tag{4.20}$$

$$\text{s.to} \quad \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^2 \leq q_i^t \cdot f_i^t \quad \forall i \in F, t \in T \tag{4.21}$$

$$\left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^4 \leq (f_i^t)^2 \cdot \delta_i^t \cdot 1 \quad \forall i \in F, t \in T \tag{4.22}$$

$$(f_i^t)^4 \leq \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^2 \cdot \phi_i^t \cdot f_i^t \quad \forall i \in F, t \in T \tag{4.23}$$

$$(f_i^t)^2 \leq v_i^t \cdot \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^2 \quad \forall i \in F, t \in T \tag{4.24}$$

$$d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \geq F_X^{-1}(\gamma_{ij}) \quad \forall i \in F, j \in P_i \tag{4.25}$$

$$(4.2)-(4.5), (4.7)-(4.14)$$

### 4.3 Summary

This chapter is devoted to mathematical formulation of our problem. First, we present mathematical model and then explain the challenges of the problem to solve. Afterwards, we continue with the reformulation of the model in which chance constraints are written in closed form expression and the second order conic reformulation of fuel consumption and CO2 emission cost function is presented.

# Chapter 5

## Heuristic Algorithms

Although integrated robust airline scheduling, aircraft fleetings and routing model can solve small instances, due to large scale parameters of the model, there are numerical stability problems also large size instances take much more time to be solved. Therefore, two heuristic methods are proposed.

### 5.1 Discretized Approximation and Cruise Speed Control Algorithm

To solve our problem faster than the integrated model, we propose discretized approximation model (hereafter DAM) which excludes nonlinearity caused by the cost function of fuel consumption and CO2 emission.

DAM is a mixed integer programming model which solves robust airline scheduling, aircraft fleetings and routing problems simultaneously. As distinct from the integrated model, in DAM, cruise time can take only a predetermined value from an interval rather than any value from that interval. For example, while cruise time of a flight in the integrated model can take any value between 85 and 115 minutes, in DAM the cruise time can be in increments of five minutes such as 85, 90, 95, 100, 105, 110 and 115. In this way, instead of the nonlinear cost function, we calculate the price of each cruise time option and use linear

terms to denote the cost related to fuel in the objective function. Hence, DAM can be solved faster than the integrated model.

In order to adapt these changes, we introduce the following parameters:

- $crs_{ik}^t$  : The  $k^{th}$  cruise time option of flight  $i$  when it is performed by aircraft  $t$ .
- $cost_{ik}^t$  : The price of  $crs_{ik}^t$  which is equal to  $(c_{fuel} + c_{CO_2}) \cdot F(crs_{ik}^t)$

Moreover, a set of binary variables for each flight and aircraft are introduced.

$\sigma_{ik}^t$ : 1 if cruise time of flight  $i$  takes the  $k^{th}$  value for aircraft  $t$ .

The formulation of the discretized approximation model is as follows:

$$\min \sum_{t \in T} \sum_{i \in F} \left( \sum_{j \in U^i} x_{ji}^t + y_i^t \right) \cdot Cspl_i \cdot \max(0, Dem_i - Cap_t) + \quad (5.1)$$

$$\sum_{i \in F} \sum_{t \in T} \sum_{k=1}^p \sigma_{ik}^t \cdot cost_{ik}^t + \sum_{i \in F} \sum_{t \in T} y_i^t \cdot Daily_t + \sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t$$

$$\text{s.to } \sum_{k=1}^p \sigma_{ik}^t = \left( y_i^t + \sum_{j \in U^i} x_{ji}^t \right) \quad \forall i \in F, t \in T \quad (5.2)$$

$$\sum_{k=1}^p \sigma_{ik}^t \cdot crs_{ik}^t = f_i^t \quad \forall i \in F, t \in T \quad (5.3)$$

$$(4.2)-(4.5), (4.7)-(4.14), (4.25)$$

In DAM, the objective function is the sum of spill cost, idle time cost, daily usage cost and the cost of fuel consumption and CO2 emission. As different from the integrated model, the cost of fuel consumption and CO2 emission is represented by the linear term  $\sum_{i \in F} \sum_{t \in T} \sum_{k=1}^p \sigma_{ik}^t \cdot cost_{ik}^t$ . Constraints (5.2) and (5.3) ensure that when flight  $i$  is performed by aircraft  $t$ , the cruise time of flight takes one of the cruise time value options. The remaining constraints are the same constraint in the integrated model.

We get a solution to robust airline scheduling, aircraft fleetling and routing problem with DAM. However, since we discretized cruise time in DAM, there is still a chance to improve that solution by considering continuous value of cruise

time. Therefore, after fixing fleeting and routing with DAM, in order to decide continuous values of cruise time, we solve the cruise speed control model (hereafter CSCM) which is suggested by Duran et al. [12]. CSCM is a nonlinear second order cone programming model which solves robust airline scheduling problem by considering departure timing, cruise time control and idle time insertion. Since the fleeting and routing decisions are made by DAM, CSCM only deals with continuous decision variables. Hence, even if it is a nonlinear model it can be solved faster than the integrated model.

In order to use the fleeting and routing generated by DAM as parameters of CSCM, we introduce the following notation.

$\bar{w}_i^t$  : A binary parameter that is 1 if flight  $i$  is performed by aircraft  $t$  in the solution of DAM

$\bar{A}$ : Set of flights  $(i, j)$  such that  $(i, j)$  are consecutive flights performed by the same aircraft in the solution of DAM

The formulation of CSCM is as follows:

$$\min \sum_{i \in F} \sum_{t \in T} \bar{w}_i^t \cdot ((c_{fuel} + c_{CO_2})F(f_i^t) + s_i^t \cdot Idle_t) \quad (5.4)$$

$$\text{s.to } x_j - x_i - TA_{ij} - \sum_{t \in T} \bar{w}_i^t \cdot f_i^t - E[A_i] - \sum_{t \in T} \bar{w}_i^t \cdot s_i^t = 0 \quad \forall (i, j) \in \bar{A} \quad (5.5)$$

$$f_i^l \cdot \bar{w}_i^t \leq f_i^t \leq f_i^u \cdot \bar{w}_i^t \quad \forall i \in F, t \in T \quad (5.6)$$

$$s_i^t \leq M \cdot \bar{w}_i^t \quad \forall i \in F, t \in T \quad (5.7)$$

$$(4.7), (4.11), (4.12), (4.25)$$

In the objective of CSCM, the costs of idle time insertion and fuel consumption and CO2 emission are minimized. Constraint (5.5) ensures aircraft connection for the routing generated by DAM. If aircraft  $t$  performs flight  $i$  as a result of DAM, constraint (5.6) keeps the cruise time of each flight between the upper and lower limits, otherwise  $f_i^t$  value is set to zero. Similarly, if aircraft  $t$  performs flight  $i$  as a result of DAM, then  $s_i^t$  can be nonnegative, otherwise it is set to zero

by constraint (5.7).

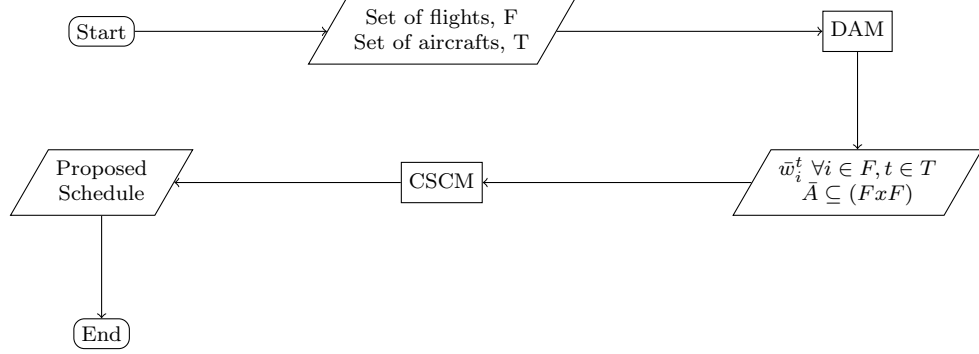


Figure 5.1: Discretized approximation and cruise speed control algorithm

In discretized approximation and cruise speed control algorithm (hereafter heuristic1), we solve two models sequentially. In Figure 5.1, we illustrate the flow chart of the algorithm. Initially we solve DAM and fix fleetings and routing, then we solve CSCM in order to find the minimum cost of idle time and fuel consumption and CO2 emission for a given fleetings and routing. In this way, instead of solving the integrated model which is a nonlinear MIP model; as a heuristic method we propose to solve first a MIP model, DAM and then a nonlinear model, CSCM whose total solution time is quite smaller than the integrated model as discussed earlier.

## 5.2 Multi-Stage Triplet Search Algorithm

As another observation, when we remove the cost terms related to the continuous decision variables from the objective function of the integrated model, we observe that solution time decreases drastically. This is due to two reasons. First, by removing fuel consumption and CO2 emission costs, we exclude nonlinearity and get a MIP model as in the case of DAM. Second, when we remove the cost of the idle time insertion as well as the cost related to fuel, aircraft fleetings and routing decisions are given more easily. To elaborate, without idle time and fuel consumption and CO2 emission costs the model only considers the trade off between daily usage and spill cost so it neglects the costs of idle time insertion

and cruise time change. Hence, we introduce daily usage and spill costs model (hereafter DSCM) in which the cost of idle time insertion and fuel consumption and CO2 emission costs are removed. All constraints of the integrated model in Section 4.1 are valid in DSCM, however the cost terms  $\sum_{i \in F} \sum_{t \in T} (c_{fuel} + c_{CO_2}) \cdot F_i^t(f_i^t)$  and  $\sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t$  are extracted from the objective function.

Although we get a non-dominated solution regarding daily usage and spill costs by DSCM, the generated solutions might have high cost of idle time and fuel since DSCM takes idle time and fuel as free. In order to enhance a solution generated by DSCM, the cost of fuel and idle time should be taken into account. Thus, we take the solution of DSCM and then by fixing fleetings and routing of DSCM, we solve CSCM which is presented in the previous section.

Even if solving CSCM after DSCM can improve the cost of idle time and fuel, this improvement is local since CSCM is solved within the fixed fleetings and routing of DSCM. By driving DSCM to generate fleetings and routing which are also favorable regarding the costs of idle time and fuel, the sequential solution of DSCM and CSCM can find better solutions. For this purpose, initially we introduce triplet which is a constituent of a fleetings and routing in an airline schedule, then present a search algorithm over triplets to improve the solutions generated by the sequential solution of DSCM and CSCM.

**Definition 1.** *A triplet,  $(i, j, t)$  is a collection of two consecutive flights  $i$  and  $j$  and the aircraft  $t$  which performs them. A triplet has the cruise times and idle time information regarding the flights in the triplet.*

The cost of a triplet is calculated as follows: the fuel consumption and CO2 emission costs are calculated for two flights and the idle time cost of the aircraft between these flights are summed and then the minimum required fuel consumption and CO2 emission costs for these flights are extracted. The minimum required fuel consumption and CO2 emission costs are calculated by assuming these flight are performed by the most efficient aircraft in the set of aircraft at optimum speed. Hence the cost of each triplet can be thought as the improvement capacity on the cost of that triplet.

In DSCM, each decision variable  $x_{ij}^t$  refers to a triplet so by fixing  $x_{ij}^t = 0$  beforehand, we avoid DSCM to have triplet  $(i, j, t)$  in the generated solution. In this manner, we present multi-stage triplet search algorithm in which we fix a variable corresponding to a triplet and solve DSCM and CSCM sequentially.

---

**Algorithm 1:** Two Stage Algorithm

---

**Input:** Triplet  $(i, j, t)$ , Set of Aircraft  $T$ , Set of Flights  $F$

- 1 Fix corresponding  $x_{ij}^t = 0$  for the given triplet  $(i, j, t)$  ;
- 2 Solve DSCM;
- 3 Set fleeting and routing;
- 4 Solve CSCM;
- 5 **if** *The problem is feasible* **then**
  - | **Output:** The generated schedule
- 6 **else**
  - | **Output:** null
- 7 **end**

---

Before multi-stage triplet search algorithm, we propose two stage algorithm which is used in every node of multi-stage triplet search algorithm in Algorithm 1. Two stage algorithm takes a set of aircraft and a set of flight and a triplet as input. In two stage algorithm, if it is feasible, a schedule is generated by sequential solution of DSCM and CSCM as discussed earlier. However, this solution is not allowed to have the given triplet by fixing variable  $x_{ij}^t = 0$  before the solution of DSCM. If a triplet is not given as input, then without any variable fixing DSCM and CSCM are solved sequentially.

In Heuristic 2, multi-stage triplet search algorithm is illustrated. Algorithm takes a set of aircraft and a set of flights with given beam size  $b$  and depth size  $d$  as input. As initialization, at the root node, we apply two stage algorithm without a triplet and we get a solution; to proceed, root node solution is recorded and the set of triplets,  $S$  is generated from the root node solution. At this step initialization is completed and a procedure which repeats itself for each depth starts. At the beginning of each depth, we have a set of triplets coming from the previous step,  $S$ , note that for the first depth, set of triplets comes from the root node solution. From  $S$ , we choose first  $b$  triplets with highest cost and then clear it. For each chosen triplet,  $b$  times at total for a depth, we apply two stage algorithm to get a solution and add the generated triplets to  $S$  for the next



---

**Heuristic 2: Multi-Stage Triplet Search Algorithm**

---

**Input:** Set of Aircrafts  $T$ , Set of Flights  $F$ , Beam Size  $b$ , Depth Size  $d$

- 1 Two stage algorithm (null,  $T$ ,  $F$ ) ;
- 2 Record the output solution of two stage algorithm to the set of solutions;
- 3 Generate the set of triplets from that solution, say  $S$ ;
- 4  $l = 0$ ;
- 5 **while** *a feasible solution exists or  $l \leq d$*  **do**
- 6     Choose first  $b$  triplets with highest cost from  $S$ ;
- 7     Clear  $S$ ;
- 8      $k = 0$ ;
- 9     **while**  $k \leq b$  **do**
- 10         Two stage algorithm ( $k^{th}$  triplet,  $T$ ,  $F$ ) ;
- 11         Record the generated solution to the set of solutions ;
- 12         Add generated triplets from two stage algorithm solution to  $S$ ;
- 13          $k = k + 1$  ;
- 14     **end**
- 15      $l = l + 1$ ;
- 16 **end**

**Output:** The best solution from the set of solutions

---

depth. We apply this procedure  $d$  times unless all two stage algorithm returns null due to infeasibility. We get one solution from the root node and  $b$  solution at each depth so total number of generated feasible solutions during the multi-stage triplet search algorithm are calculated as follows:

$$\text{Solution Number} = b \cdot d + 1 \tag{5.8}$$

Eventually the best solution among  $b \cdot d + 1$  solutions is proposed.

### 5.2.1 Numerical Example

In order to elaborate how multi-stage triplet search algorithm works, we present a small numerical example. Here, we use 23 flight network, for service level 75%. In this example, we take beam size  $b = 3$  and depth size  $d = 5$ . We get 16 solutions for this instance by equation (5.8).

In Figure 5.2, an example illustration of search tree is shown for beam size

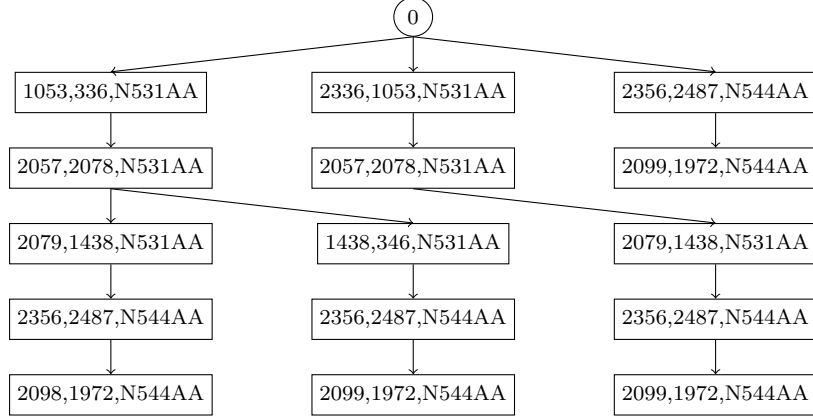


Figure 5.2: Multi-Stage Triplet Search, Beam Size  $b = 3$ , Depth Size  $d = 5$

$b = 3$  and depth size  $d = 5$ . We elaborate how the algorithm works with this example. Initially at root node, within the two stage algorithm, two models are solved sequentially and the set of triplets from the root node solution is generated. From this set of triplets the three triplets with highest cost are chosen, which are (1053,336,N531AA), (2336,1053,N531AA) and (2356,2487,N544AA). Their costs are calculated as explained previously. At the first depth, we fix the corresponding variable, in DSCM, i.e.  $x_{ij}^t = 0$  and avoid to generate the associated triplets for each node. For example, for the first node, DSCM has the constraint  $x_{ij}^t = 0$  for flights 1053, 336 and aircraft N531AA. By the same reasoning, at depth one, three times two models are solved sequentially in which various triplets are avoided for each node. After getting three solutions, the set of triplets are filled with the triplets generated from those three solutions. Then again the three triplets with highest cost from that set are chosen and depth two starts. Similar to the first depth, at depth two again three times two stage algorithm is applied, however, this time the triplet constraint at the previous node is added as well as the associated triplet constraint of that node. For example, in the first node of the second depth, the associated triplet is (2057, 2078, N531AA) however due its previous node the decision variable associated to the (1053,336, N531AA) are fixed to zero as well. In case depth size is reached or all the nodes are infeasible the search algorithm stops.

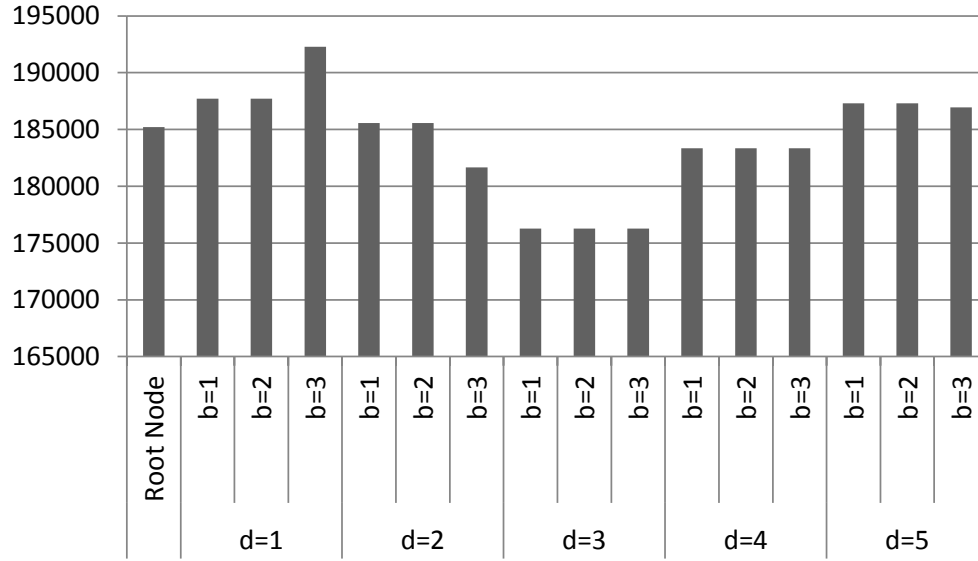


Figure 5.3: Total costs of the solutions

In Figure 5.3, the cost of each solution generated during the triplet search is shown. As it can be seen, there is not a pattern in the changes on the costs of the generated solutions depending on the depth. The reason is that Multi-Stage Triplet Search Algorithm tackles the cost terms of the problem separately; while a step of the algorithm can increase some cost terms, it can decrease the other cost terms so total cost can increase or decrease. To elaborate, the routings and aircraft assignments are determined by minimizing the sum of daily usage and spill costs without considering other cost terms, and similarly triplets are chosen in order to decrease the cost of idle time, fuel consumption and CO2 emission costs regardless of the daily usage and spill costs.

### 5.3 Summary

In this chapter, our heuristic methods which are proposed to solve larger problems faster than the integrated model are explained in detail. First, discrete approximation and cruise speed control algorithm is explained. In this algorithm, two mathematical models, DAM and CSCM are solved sequentially. Second, multi-stage triplet search algorithm is explained. In multi-stage triplet search algorithm, we define triplet concept and conduct a search algorithm over triplets by applying two stage algorithm. In two stage algorithm, we first fix a decision variable related to a triplet to zero and solve DSCM, CSCM models sequentially. Eventually, we present a numerical example to elaborate how multi-stage triplet search algorithm works.

# Chapter 6

## Computational Study

In this study, we present the integrated robust airline scheduling, aircraft fleet and routing problem by incorporating cruise time control within consideration of passengers' service levels and maintenance requirements. In order to solve this problem, we propose a nonlinear MIP model. Although this integrated model can solve small samples in reasonable time, it takes too much time to solve large size problems and also we observe some numerical stability problems due to very large and very small parameter values in the integrated model as sample size increases. Therefore, we propose two heuristic methods as alternatives to the integrated model. In this section, we compare the performances of these three solution methods in terms of different airline cost components against to the published schedule. Meanwhile, the performance of these methods, such as regarding CPU time, are compared among each other.

### 6.1 Experimental Design

In this computational study, we make a  $2^k$  full-factorial experimental design. There are four experimental factors and their corresponding levels are given in Table 6.1.

The first factor is the fuel cost which is the price of jet fuel per ton. In a recent

Table 6.1: Factor values

Factors	Description	Levels		
		Low(0)	Medium(1)	High(2)
A	Fuel Cost	\$600	-	\$1200
B	Base Spill Cost	\$15	-	\$60
C	$\nu$	1	-	10
D	Service Level	75%	85%	95%

study conducted by Duran et al. [12], the fuel prices are taken as \$1.8/gallon for the lower setting and \$3.6 gallon for the higher setting. The same factor values for the fuel cost are adopted also in this study.

The second factor is the base spill cost. In order to experiment on the effect of the spill cost, we choose the base spill cost which is used as a multiplier in the spill cost of a flight per passenger. Spill cost of a flight per passenger is calculated within the consideration of the congestion coefficients of the origin and destination airports. When a passenger is spilled from a flight which is between two congested, i.e. with a high number of visiting passengers, airports, the spill cost of that passenger becomes more. The reason behind is that this passenger flies from or to the airports with high market demand. For each flight, the spill cost per passenger is calculated as follows.

$$Csp_i = BaseSpillCost \cdot (e_{Or_i}) \cdot (e_{Dn_i}) \quad (6.1)$$

The third experimental factor,  $\nu$ , is used for the calculation of the daily usage cost of an aircraft. In our study, we consider the daily usage cost of an aircraft which refers to the sum of the fixed operating costs and the lost opportunity cost when that aircrafts is used as discussed in Chapter 3. We calculate daily usage cost related to the unit idle time cost since it is rational that a valuable aircraft has both high unit idle time cost and daily usage cost. For each aircraft, the daily usage cost is calculated as follows.

$$Daily_t = \nu \cdot Idle_t \cdot 60 \quad (6.2)$$

The last factor is the service level of the passengers connection. As distinct from the first three factors, this factor has three levels. While the first three factors have linear effects on the objective function, the effect of service level on the objective function is not as straightforward as the other factors since it is not a direct cost term of the objective function. Hence in order to reveal its effect in the experimental design, we choose to adopt three levels on this factor.

In this study for a flight network, we take 72 randomly generated runs which is the sum of three replication of random parameters. Each replication has 24 instances which come from  $2 \cdot 2 \cdot 2 \cdot 3$  due to two levels of the first three factors and three levels of the last factor.

## 6.2 Input Data

In this study, we use the flight network which is in the work of Aktürk et al. [38]. The published schedule is provided in Table 6.2. The flight information were taken from the BTS database [43]. In the published schedule, each column represents the tail number, flight number, departure and arrival airport, departure time, flight block time and arrival time of each flight respectively. From the published schedule, we take the set of flights and set of aircrafts as input to our proposed solutions.

Among these 114 flights, we generate three different flight networks. The first one is 23 flight network; the second is 35 flight network and the last one is 114 flight network as given in the Table 6.2. We conduct all computational study over these flight networks. In order to illustrate the details of the flight networks, the statistics after pre-solve operation of the integrated model are presented in Table 6.3.

As stated in Chapter 3, the input data regarding each aircraft in our study is fuel consumption and CO2 emission cost parameters, seat capacity, idle time cost, limit on total cruise time and the airport on which it has to land to ensure maintenance requirements and the first airport from which it could fly; lastly the cost incurred when that aircraft is used on that day. There are six different

Table 6.2: Published schedule for 114 flight network

Tail #	Flight #	From	To	Dep. Time	Dur.	Tail #	Flight #	From	To	Dep. Time	Dur.
N530AA	398	ORD	LGA	06:15	134	N3ETAA	1704	ORD	EWR	06:35	125
	319	LGA	ORD	09:25	170		1883	EWR	ORD	09:30	160
	2329	ORD	DFW	13:35	155		810	ORD	DCA	13:10	105
	2364	DFW	ORD	17:00	150		2013	DCA	ORD	15:45	135
N459AA	394	ORD	LGA	06:50	135		2013	ORD	LAS	19:00	250
	321	LGA	ORD	10:00	170	N3DYAA	1063	ORD	LAX	08:50	275
	366	ORD	LGA	13:55	140		874	LAX	ORD	14:30	255
	347	LGA	ORD	17:15	170		874	ORD	BOS	19:45	135
N531AA	2303	ORD	DFW	06:45	155	N3DRAA	1021	ORD	LAS	08:30	245
	2336	DFW	ORD	10:10	140		1544	LAS	ORD	13:25	215
	1053	ORD	AUS	13:25	170		1544	ORD	DCA	18:00	105
	336	AUS	ORD	17:00	165	N5DXAA	1048	ORD	MIA	07:35	190
	336	ORD	LGA	20:40	125		1763	MIA	ORD	11:55	200
N4XGAA	2079	ORD	SAN	08:45	270		1899	ORD	MIA	16:20	185
	1438	SAN	ORD	14:00	250	N454AA	2441	ORD	ATL	06:30	120
	346	ORD	LGA	19:50	135		1986	ATL	ORD	09:15	135
N598AA	1341	ORD	SFO	07:50	295		1872	ORD	MCO	12:25	160
	348	SFO	ORD	13:30	265		1131	MCO	ORD	15:50	185
	1521	ORD	TUS	19:15	235	N4YMAA	1137	ORD	MSY	08:20	145
N439AA	2455	ORD	PHX	07:10	240		1768	MSY	ORD	11:30	150
	358	PHX	ORD	11:55	210		1768	ORD	PHL	15:05	125
	358	ORD	LGA	16:25	145		1697	PHL	ORD	18:00	155
	371	LGA	ORD	20:00	155	N467AA	1823	ORD	PBI	09:20	175
N475AA	407	ORD	STL	06:20	70		2067	PBI	ORD	13:00	200
	755	STL	ORD	08:35	75		2067	ORD	STL	17:15	70
	755	ORD	SAT	10:45	180		1186	STL	ORD	19:10	80
	408	SAT	ORD	14:30	160	N536AA	2305	ORD	DFW	07:45	160
	408	ORD	PHL	18:05	125		2344	DFW	ORD	11:35	140
N3EEAA	876	ORD	BOS	06:35	130		1201	ORD	STL	14:50	65
	413	BOS	ORD	09:35	185		1815	STL	ORD	17:00	80
	413	ORD	SNA	13:45	275		1815	ORD	SLC	19:15	270
	1262	SNA	ORD	19:10	230	N420AA	1686	ORD	RDU	06:50	110
N4YDAA	451	ORD	SFO	09:45	295		2435	RDU	ORD	09:25	135
	554	SFO	ORD	15:45	265		2435	ORD	PHX	12:35	235
	496	ORD	DCA	06:45	100		1206	PHX	ORD	17:15	205
N3ERAA	1715	DCA	ORD	09:15	130	N546AA	1462	ORD	EWR	08:00	140
	1715	ORD	LAS	12:25	255		1387	EWR	ORD	11:25	160
	1708	LAS	ORD	17:20	220		1397	ORD	MCO	15:00	160
N5CLAA	1425	ORD	SNA	08:25	280		1221	MCO	ORD	18:25	175
	556	SNA	ORD	14:00	240	N4WPAA	2311	ORD	DFW	09:05	155
	1940	ORD	MIA	19:25	180		2348	DFW	ORD	12:35	140
N535AA	2460	ORD	RSW	06:45	165		1797	ORD	STL	15:50	70
	564	RSW	ORD	10:20	185		1982	STL	ORD	18:00	80
	1446	ORD	EWR	14:55	165		1339	ORD	SAN	20:15	270
	1411	EWR	ORD	18:45	165	N5EBAA	2375	ORD	EGE	08:10	175
N3DMAA	568	ORD	FLL	07:25	175		2378	EGE	ORD	12:25	165
	711	FLL	ORD	11:10	195		1677	ORD	SNA	18:40	270
	2021	ORD	SJU	15:25	275		2099	ORD	LAX	07:00	270
N544AA	2463	ORD	MCI	06:25	90	N3DUAA	1972	LAX	ORD	12:40	245
	754	MCI	ORD	08:40	90		1972	ORD	RDU	17:45	115
	2321	ORD	DFW	11:15	155	N3ELAA	2057	ORD	SJU	08:30	290
	2356	DFW	ORD	14:40	140		2078	SJU	ORD	14:25	335
	2487	ORD	DEN	17:50	165	N3DTAA	2363	ORD	HDN	09:50	170
N3EBAA	1565	ORD	MSP	06:40	90		2318	HDN	ORD	13:40	170
	779	MSP	ORD	09:00	85	N412AA	2345	ORD	DFW	17:15	155
	779	ORD	SAN	11:35	260		2374	DFW	ORD	20:40	130
	1358	SAN	ORD	16:45	235						
	1358	ORD	BOS	21:50	125						



Table 6.3: Problem size with different instances

Flights	Aircraft	Rows	Columns	Binary Var.	Quadratic Con.	Indicators
23	7	9190	10249	5303	966	482
35	10	23782	26353	15632	2100	1024
114	31	498807	545734	436764	21204	10853

aircraft types which is used in the published schedule. The attributes of each aircraft type are shown in Table 6.4. These parameters are taken from BADA (EUROCONTROL) [40] and they are used to calculate fuel consumption of each flight depending on the performing aircraft. After fuel consumption is calculated, in order to find the cost of fuel consumption and CO2 emission we multiply unit fuel cost per ton as shown previously in Table 6.1 as factor A. For the start and end airport for each aircraft, we take the first and last airports of that particular flight. For example, in the published schedule the aircraft N530AA performs its first flight from ORD airport and it lands to ORD at the end of the day; in our study we take ORD as the start and end airport of N530AA. We take the limit on the total cruise time as 720 min. for each aircraft.

Table 6.4: Aircraft parameters

Aircraft type	B727 228	B737 500	MD 83	A320 111	A320 212	B767 300
Capacity	134	122	148	172	180	218
Mass (kgs)	74000	50000	61200	62000	64000	135000
Surface(m2)	157.9	105.4	118	122.4	122.6	283.3
$C_{D0,CR}$	0.018	0.018	0.0211	0.024	0.024	0.021
$C_{D2,CR}$	0.06	0.055	0.0468	0.0375	0.0375	0.049
$C_{f1}$	0.53178	0.46	0.7462	0.94	0.94	0.763
$C_{f2}$	276.72	300	638.59	50000	100000	1430
$C_{fcr}$	0.954	1.079	0.9505	1.095	1.06	1.0347
MRC speed	867.6	859.2	867.6	855.15	868.79	876.70
Base Turntime	32	36	26	28	30	40
Idle Time Cost(\$)	150	140	142	136	144	147

In Table 6.5, aircraft type of each individual tail number in 114 flight network is presented.

Moreover, the input data regarding each flight in our study is origin airport, destination airport, cruise time interval, departure time interval, demand and the spill cost. Origin and destination airports are taken from the published schedule directly. For the cruise time interval, first we take the original cruise time of

Table 6.5: Aircraft type

Tail #	Aircraft Type	Tail #	Aircraft Type
N531AA	B767 300	N3DMAA	B737 500
N598AA	MD 83	N544AA	B767 300
N475AA	MD 83	N3EBAA	B737 500
N3EEAA	A320 111	N3ETAA	A320 111
N4YDAA	MD 83	N3DYAA	A320 111
N3ERAA	A320 111	N5DXAA	B727 228
N5CLAA	B767 300	N454AA	A320 212
N535AA	B727 228	N4YMAA	A320 212
N3DRAA	B737 500	N420AA	A320 212
N467AA	A320 212	N546AA	B767 300
N3DTAA	A320 111	N4WPAA	B737 500
N412AA	B737 500	N439AA	A320 212
N530AA	B767 300	N5EBAA	B767 300
N459AA	A320 212	N4EBAA	MD 83
N4XGAA	A320 212	N3DUAA	MD 83
N536AA	B767 300	N3ELAA	B727 228

each flight by subtracting 20 min. from each flight duration. Afterwards for the upper and lower limit on cruise time, we take 85% and 115% of the original cruise time, since Delgado and Prats [44], state that the cruise speed can be varied by around 10% from max-range cruise speed. For the departure time interval, we allowed departure of each flight 15 min. earlier or later than the published departure time. Flight demands are taken randomly as in the study by Şafak [28] for different replications. Lastly, spill costs are explained previously in factor values.

In addition to flights and aircrafts, there is a need for the passenger connection time and airport congestion coefficients as input data. Passenger connection times are taken uniformly between 25 and 40 min. Passenger connections are possible between two flights  $i$  and  $j$ , if the original departure time of flight  $j$  is within 30 minutes or 180 minutes of the original arrival time of flight  $i$  and destination airport of flight  $i$  is same as the origin airport of the flight  $j$ . Airport congestion coefficients and turntimes of the aircrafts are taken from the study of Duran et al. [12].

$$TA_{ij}^t = BaseTurntime^t \cdot (e_{Dn_i}) \quad (6.3)$$

Table 6.6: Congestion coefficients

Airport	Location	Coefficient	Airport	Location	Coefficient
MIA	Miami, FL	1.40	DCA	Washington, DC	1.08
ORD	Chicago, IL	1.37	SAN	San Diego, CA	1.05
LAX	Los Angeles, CA	1.35	STL	St.Louis, MO	1.05
DEN	Denver, CO	1.35	MCI	Kansas City, MO	1.02
DFW	Dallas, TX	1.32	AUS	Austin, TX	1.00
LGA	New York, NY	1.30	RDU	Raleigh/Durham, NC	1.00
BOS	Boston, MA	1.30	MSY	New Orleans, LA	0.98
ATL	Atlanta, GA	1.28	SNA	Santa Ana, CA	0.98
PHX	Phoenix, AZ	1.25	SAT	San Antonio, TX	0.95
LAS	Las Vegas, NV	1.25	RSW	Fort Myers, FL	0.95
SFO	San Francisco, CA	1.20	SJU	San Juan, PR	0.92
MSP	Minneapolis, MN	1.15	PBI	West Palm Beach, FL	0.90
PHL	Philadelphia, PA	1.15	TUS	Tuscan, AZ	0.88
EWR	Newark, NJ	1.12	MCO	Orlando, FL	0.85
FLL	Fort Lauderdale, FL	1.12	EGE	Eagle, CO	0.85
SLC	Salt Lake City, UT	1.08	HDN	Hayden, CO	0.80

### 6.3 Analysis on the Integrated Model

We start the analysis on the results of our computational study by pointing out the significance of our integrated approach. For this purpose, we compare the cost of the published schedule which is generated by the sequential approach and the proposed schedule which is the output of our integrated robust airline scheduling, aircraft fleet and routing model with cruise speed control. Hence, we illustrate the cost improvement for total cost and other cost terms with different factor values and replications of random input. To do this, we use the 23 flight network. The improvement is calculated as in Section 3.4.

Table 6.7: Cost improvement over the published schedule

Factors	Level	Fuel& CO2 (%)			Idle Time (%)			Daily Usage (%)			Total(%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	4.15	8.27	14.51	89.25	93.75	95.98	0	8.58	14.71	14.88	17.98	22.77
	2	4.16	9.27	14.51	89.25	93.46	95.7	0	7.35	14.71	13.38	15.99	19.79
B	0	4.16	9.33	14.51	89.25	93.38	95.7	0	7.35	14.71	14.53	17.94	22.77
	2	4.15	8.21	14.51	89.25	93.83	95.98	0	8.58	14.71	13.38	16.02	21.24
C	0	4.16	13.38	14.51	89.25	93.14	95.98	0	1.23	14.71	13.74	18.85	22.77
	2	4.15	4.15	4.15	91.78	94.07	95.70	14.71	14.71	14.71	13.38	15.11	16.55
D	0	4.15	8.80	14.51	92.19	94.59	95.98	0	7.97	14.71	13.46	17.08	22.77
	1	4.16	8.76	14.51	91.29	93.92	95.37	0	7.97	14.71	13.44	17.01	22.63
All Instances	2	4.16	8.76	14.51	89.25	92.30	93.78	0	7.97	14.71	13.38	16.86	22.30
		4.15	8.77	14.51	89.25	93.61	95.98	0	7.97	14.71	13.38	16.98	22.77

As it is shown in Table 6.7, our integrated model provides around 17% average improvement on the total cost of the published schedule over all factor levels. Moreover, from Table 6.7, the trade-off between daily usage cost and fuel consumption & CO2 emission costs can be deduced. When daily usage cost is

high, the improvement over the fuel & CO2 emission costs is limited around 4% at maximum and the cost improvement over daily usage cost is around 14% at minimum. That means in order to decrease total number of aircraft used, the model compresses the cruise time of flights and causes less improvement on the costs related to fuel. Similarly, when the cost of fuel consumption and CO2 emission is high, the average improvement on the daily usage cost decreases from 9% to 7% approximately.

### 6.3.1 The effect of cruise speed control

We continue the analysis on the results of our computational study by indicating what would happen without cruise time control, in order to emphasize its contribution explained in Section 1.2. By this means, we present the analysis on the integrated model and compare its performance with other integrated models which do not consider cruise speed control.

For this purpose, we consider two models. Both models are integrated models in which robust airline scheduling, aircraft fleetings and routing decisions are made within the consideration of passengers' service level and maintenance requirements. The first model tries to minimize the sum of spill cost, fuel consumption and CO2 emission cost, idle time cost and daily usage cost without cruise time control. The second one tries to minimize sum of spill cost, idle time cost and daily usage cost but considers neither fuel consumption or CO2 emissions cost nor cruise time control. Hereafter, Alternative1 refers to the model which considers fuel and CO2 emission cost but not cruise time control and Alternative2 refers to the model which considers neither fuel or CO2 emissions cost nor cruise time control. We present the improvement of our integrated model on the total cost for Alternative1 and Alternative2. We calculate the improvement by the following equation.

$$\text{Improvement} = \frac{\text{Cost of Alternative} - \text{Cost of Our Model}}{\text{Cost of Alternative}} \quad (6.4)$$

In Table 6.8, the improvements provided by the cruise time control over 23

Table 6.8: Total cost improvement of cruise speed control

Factors	Level	Imp. on Alt1 (%)			Imp. on Alt2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	3.9	8.8	12.4	4.9	9.9	13.0
	2	4.7	7.9	10.4	7.7	11.2	13.1
B	0	4.7	8.2	12.4	7.7	10.6	13.0
	2	3.9	8.5	12.1	4.9	9.2	12.3
C	0	3.9	5.7	7.7	4.9	8.1	10.8
	2	9.7	11.0	12.4	10.4	11.7	13.0
D	0	4.2	8.5	12.4	5.3	10.0	13.0
	1	4.1	8.4	12.4	5.1	10.0	13.0
	2	3.9	8.3	12.4	4.9	9.8	13.0
All Instances		3.9	8.4	12.4	4.9	9.9	13.0

flight network are shown. When all the results are considered, it is seen that cruise time control can achieve 8% and 10% average cost improvement over Alternative1 and Alternative2, respectively. When we examine the effect of different factor levels, the cost improvement of cruise time control is increasing remarkably depending on the factor C. That means, when higher utilizations of aircrafts are more valuable, cruise time control is getting more crucial. The essence of the matter is that cruise time control can decrease the total cost around 9% and this average improvement can increase up to 11% when the daily usage cost is higher.

Our approach improves the cost value of the published schedule and the other integrated approaches which do not consider cruise time control. To illustrate how long it takes and to deduce when the model is solved faster, in Table 6.9 we present the CPU time analysis for the 23 flight network.

As it is seen in Table 6.9, the CPU time is more than 10 minutes in average. It is observed that at the factor levels in which fuel & CO2 emission cost is dominated by the other cost factors, CPU time is quite small. The reason behind is that probably the indicator variables on routing and fleeting can take values faster so the solver can handle the problem as a nonlinear problem rather than a nonlinear MIP problem. However, when fuel & CO2 emission cost is not dominated, it is possible to see CPU time around 1 hour.

Table 6.9: CPU time analysis

Factors	Level	CPU Time in Seconds		
		Min.	Avg.	Max.
A	0	18	883	4048
	2	375	1054	2216
B	0	606	1171	2616
	2	18	766	4048
C	0	465	1240	4048
	2	18	697	2616
D	0	22	938	2216
	1	18	867	1920
	2	51	1099	4048
All Instances		18	968	4048

## 6.4 Analysis on the Heuristic Methods

In order to solve, larger instances in reasonable times, we propose two heuristic methods. In this section, we first compare the gap and CPU time performances of our proposed heuristic methods with the integrated model over 23 flight network. Afterwards, we conduct the same analysis over 35 flight network. However, for the solutions of the integrated model over 35 flight network, we limit solution time to 5400 seconds, so in that part we compare the best solution of the integrated model in 5400 seconds with the heuristic methods. Eventually, we compare the performances of heuristic methods over 114 flight network. For 114 flight network, since we are not able to get a feasible solution in a reasonable time with integrated model, we compare the schedules generated by heuristic methods with the published schedule for 114 flight network. Eventually, we analyze the working structure of each heuristic methods. For the ease of the reader, from now on we write heuristic1 for discretized approximation and cruise speed control algorithm and heuristic2 for multi-stage triplet search algorithm. By considering the trade off between solution quality and CPU time, we take the value of the discretization parameter of heuristic1 7 for all 23, 35 and 114 flight networks while we use 5 for the beam size and 8 for the depth size of heuristic2 for 23 and 35 flight networks. Eventually for 114 flight network, we use 3 for the beam size and 10 for the depth size of heuristic2.

### 6.4.1 Performance Analysis of Heuristic Methods

In this section, we analyze the heuristic methods in terms of gap and improvement. Initially, the gap between the integrated model and the heuristic methods are examined with 23 flight network.

We calculate the gap between the optimal solution of the integrated model and the solution of heuristic methods by the following equation.

$$\text{Gap} = \frac{\text{Heuristic} - \text{Optimal}}{\text{Optimal}} \quad (6.5)$$

Table 6.10: Gap of heuristic methods over 23 flight network

Factors	Level	Heuristic1 (%)			Heuristic2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	0.000	0.000	0.001	0.000	0.3	2.1
	2	0.000	0.000	0.002	0.000	0.4	3.7
B	0	0.000	0.000	0.002	0.000	0.5	3.7
	2	0.000	0.000	0.002	0.000	0.2	2.1
C	0	0.000	0.000	0.002	0.000	0.7	3.7
	2	0.000	0.000	0.000	0.000	0.000	0.001
D	0	0.000	0.000	0.001	0.000	0.4	3.7
	1	0.000	0.000	0.000	0.000	0.3	3.7
	2	0.000	0.000	0.002	0.000	0.2	2.1
All Instances		0.000	0.000	0.002	0.000	0.3	3.7

As it seen from Table 6.10, the gaps of the heuristic methods are less than 1% on the average at all instances. It is worthwhile to point out that Heuristic1 has 0.002% maximum gap over all factor levels and all instances. Although, Heuristic2 has maximum gap 3.730% over all instances, when factors C and B are high, it has maximum gap 0.001% and 2.093% respectively. This is because Heuristic2 fixes the routings and fleeting by just considering daily usage and spill cost. When these cost terms dominate the other cost terms on routings and fleeting decisions, ignoring fuel & CO2 emission and idle time costs at routing and fleeting gets less significant. Hence Heuristic2 causes smaller gaps when factors B and C are high and factor A is low.

Afterwards, we compare the performances of heuristic methods with the integrated model in 5400 seconds over 35 flight network. Here we calculate the improvement of solutions of heuristic methods over the best solutions generated by the integrated model in 5400 seconds as follows.

$$\text{Improvement} = \frac{\text{Best Incumbent Solution} - \text{Heuristic}}{\text{Heuristic}} \quad (6.6)$$

Table 6.11: Improvement of heuristic methods over 35 flight network

		Heuristic1 (%)			Heuristic2(%)		
Factors	Level	Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	0.000	4.1	25.9	-0.041	4.0	25.9
	2	0.000	2.3	11.0	-0.291	2.1	10.9
B	0	0.000	2.8	15.0	-0.149	2.5	15.0
	2	0.000	3.6	25.9	-0.291	3.6	25.9
C	0	0.010	3.6	25.9	-0.291	3.3	25.9
	2	0.000	2.9	14.9	-0.149	2.9	14.9
D	0	0.000	3.3	14.9	-0.291	3.1	14.9
	1	0.000	4.1	25.9	-0.128	3.9	25.9
	2	0.000	2.3	13.2	-0.149	2.2	13.2
All Instances		0.000	3.2	25.9	-0.291	3.1	25.9

When we compare the best incumbent solution generated by the integrated model in 5400 seconds with the solutions generated by heuristic methods, in Table 6.11, it seen that both heuristic methods surpass the best solution of the integrated model on the average. While heuristic1 achieves 3.220%, heuristic2 achieves 3.070% average improvement over all instances. The negative improvement in heuristic2 means that the best solution generated in 5400 seconds by the integrated model has smaller objective value than heuristic2 has.

Eventually, we analyze the heuristic methods over 114 flight network. As discussed earlier, the integrated model cannot find a feasible solution in 6 hours so we compare performance of the heuristic methods over the cost of published schedule. The cost of the published schedule is calculated as in Section 3.4. Here the improvement of heuristic methods are calculated as they are in Sections 3.4 and 6.3.



Table 6.12: Improvement of heuristic methods over 114 flight network

Factors	Level	Heuristic1 (%)			Heuristic2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	10.6	17.9	25.9	10.6	16.7	25.9
	2	9.8	14.6	19.8	9.2	12.3	15.9
B	0	11.3	17.1	25.9	9.8	14.9	23.5
	2	9.8	15.4	24.5	9.2	14.1	22.7
C	0	14.6	21.0	25.9	12.2	18.3	23.5
	2	9.8	11.5	12.4	9.2	10.7	11.9
D	0	9.9	16.4	25.9	9.4	14.6	23.5
	1	9.8	16.3	25.9	9.3	14.6	23.4
	2	9.8	16.0	25.3	9.2	14.2	22.6
All Instances		9.8	16.2	25.9	9.2	14.5	23.5

In Table 6.12, it is seen that both heuristic methods provide around 15% average improvement over all instances in comparison to the published schedule. It is observed that when daily usage cost is high, the average improvement over all instances by both methods are limited to approximately 11% on average.

For constraint (4.25), we take  $\gamma_{ij}$  values equal to the levels of factor D. In this way, minimum service levels of the proposed schedule are equal to 75%, 85% and 95% while on the average, service levels are around 98%, 98% and 99%, respectively. However in the published schedule, each connection has a different service level and some of them are lower than even 75% which is the lowest level of factor D in the computational study. Specifically, for different values of  $TP_{ij}$  which are the randomly generated minimum required passenger connection times, the minimum service level of the published schedule is 73%, 2% and 76%.

When all flight networks are considered, it can be deduced that the performance of heuristic1 is better than heuristic2 in terms of gap and improvement.

Table 6.13: CPU time analysis

	Integrated Model (in sec.)			Heuristic1 (in sec.)			Heuristic2 (in sec.)		
	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
23 Flight Network	18	968	4048	0.8	1.1	1.8	10.5	12.4	15.3
35 Flight Network	4243.4	5381.1	5400	3.2	5.7	7.8	31.6	40.2	54.7
114 Flight Network	-	-	-	724.8	5173.4	5400	1702.6	3254.3	5400

Moreover, heuristic1 also performs better than heuristic2 in 23 flight network and 35 flight network regarding CPU time. However in 114 flight network, Heuristic2 produces solutions 2000 seconds faster than heuristic1 on average.

As an overall evaluation, it can be said that both heuristic methods have quite small optimality gaps over 23 flight network; heuristic methods surpass the best solution of the integrated model in 5400 seconds over 35 flight networks. Moreover heuristic methods work quite faster than the integrated model in terms of the CPU time. When the performance of the heuristic methods are compared, it can be said that heuristic1 surpasses the heuristic2 in terms of optimality gap and improvement over the best solution and published schedule over all flight networks. Even for the CPU time, heuristic1 is more favorable than heuristic2 in 23 and 35 flight networks, however in 114 flight the CPU time of heuristic2 is 2000 seconds less than the CPU time of heuristic1 on the average. In other words, in 114 flight network even if the improvement of heuristic2 is 1.8% smaller than the improvement of heuristic1, heuristic2 works 2000 seconds faster on average.

#### 6.4.2 Analysis on the structure of heuristic methods

In this section, we analyze the working structure of heuristic methods as discussed in Chapter 5. We start with heuristic1. For heuristic1, it is discussed that after DAM, due to the effect of discretization, CSCM has a potential to improve the fuel consumption and CO2 emission cost. To quantify the improvement after DAM, we calculate the proportion given by the following equation:

$$\text{Improvement} = \frac{\Delta \text{Fuel \& CO2 cost, idle time cost}}{\text{Fuel \& CO2 cost, idle time cost of DAM} - \text{Min. required fuel \& CO2 cost}} \quad (6.7)$$

In equation (6.7),  $\Delta \text{Fuel \& CO2 cost, idle time cost}$  refers to the change of the sum of the fuel & CO2 cost, idle time cost between DAM and CSCM. In denominator, we extract the minimum required fuel & CO2 cost from the fuel & CO2 cost, idle time cost of DAM since there is a minimum required fuel and CO2 cost caused by the fixed fleeting and routing coming from DAM. The purpose

behind is to evaluate the improvement of CSCM over the possible improvement capacity.

In Figure 6.1, we illustrate the improvement of CSCM over different flight networks for each run. The vertical axis represents the improvement ratio calculated by the equation (6.7) and the horizontal axis represents the runs. The average improvement is 6%, 6% and 8% over 23, 34 and 114 flight networks respectively. It is observed that the effect of CSCM is high when fuel cost is high. The average improvement when fuel cost is high, 8%, 7% and 9% over 23, 34 and 114 flight networks respectively. This is a direct implication of that the adverse effect of discretization becomes more significant when fuel cost is high.

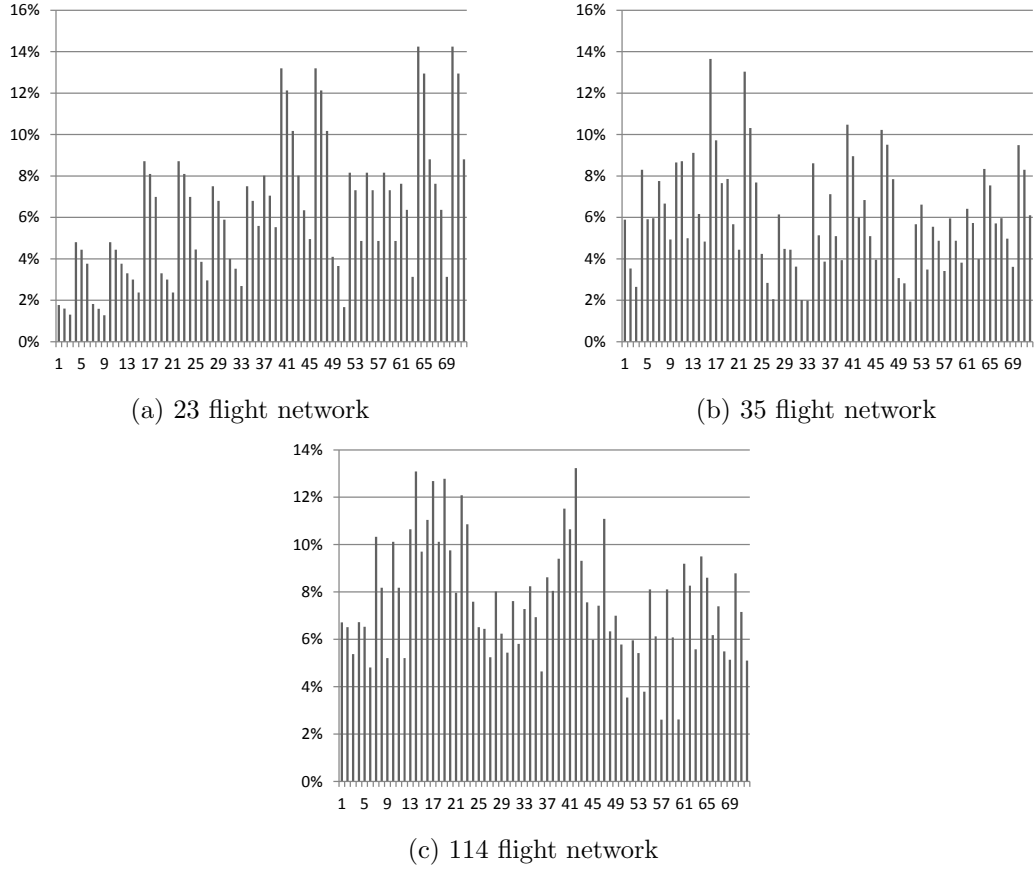


Figure 6.1: Effect of CSCM on fuel & CO2 cost and idle time cost

Then, we continue with heuristic2 and focus on the improvement after root node. In order to show the improvement after root node, in Figure 6.2, we present

the total cost decrease after root node over different flight networks. The vertical axis represents total cost change between the root node and the proposed solution while the horizontal axis represents the runs. The average cost changes after root node are \$89974, \$4618 and \$4933 over 23, 35 and 114 flight networks respectively. The runs in which the cost changes are zero, the root node solution is proposed as best solution. This situation is observed when the spill cost and daily usage cost are high and the other cost terms are low. The reason is that at the root node, the best solution regarding the sum of spill cost and daily usage cost is found since these terms dominate the other cost terms, the root node solution becomes the best solution and even it is seen in 23 flight network, the root node finds solutions which is near to the optimal solution of the integrated model when daily usage and spill costs are high. For example, in 23 flight network the average optimality gap of the solutions generated at the root node when daily usage and spill costs are high is 0.014%.

## 6.5 Summary

This chapter is devoted to the computational study dealing with the analysis and the comparison of the three proposed solution methods, which are the integrated model, heuristic1 and heuristic2, among each other and against the published schedule. For this purpose, we conduct a  $2^k$  full-factorial experimental design where four experimental factors, fuel cost, base spill cost, daily usage cost coefficient and service level, are considered. Moreover the random input data for each factor combination we generate 3 replications. Hence for each flight network we take 72 randomly generated runs which is the sum of three replication with 24 instances. Within this setting, we use 23, 35 and 114 flight networks in our analysis.

We first analyze the integrated model and compare its performance against the published schedule in 23 flight network. Moreover, we illustrate the effect of the cruise time control by comparing the integrated model with two alternative models which do not consider cruise time control.

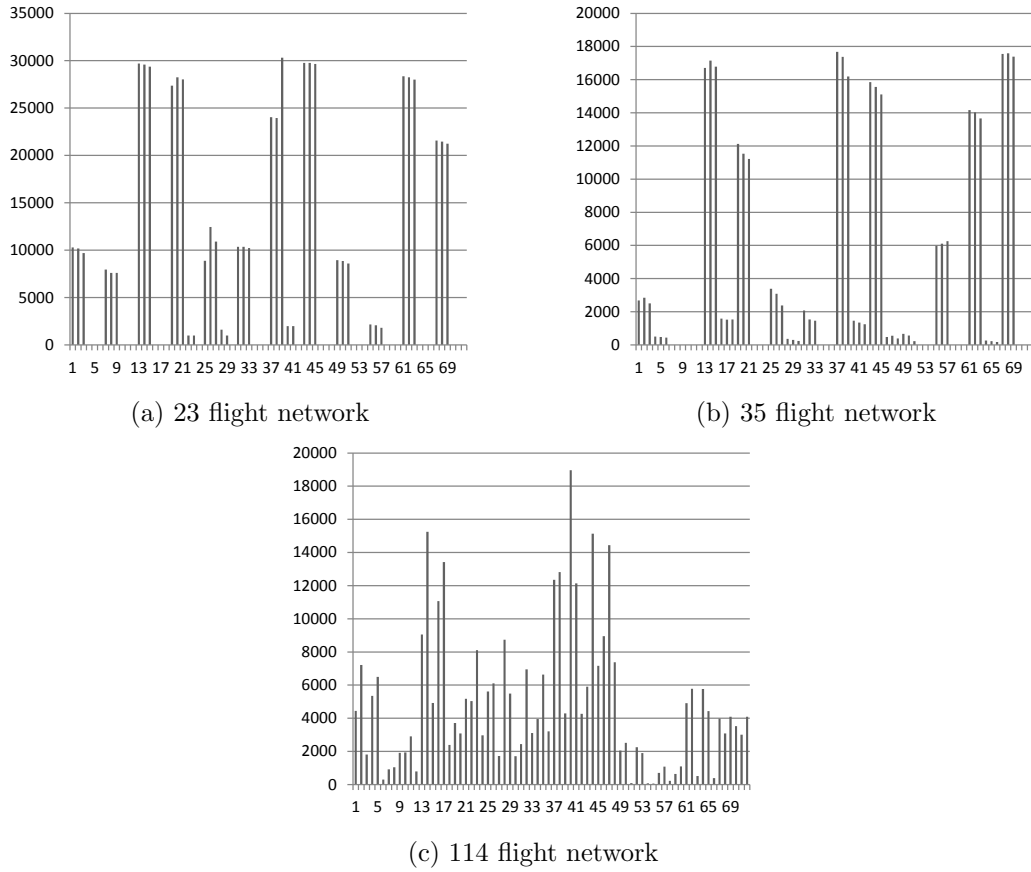


Figure 6.2: Improvement over root node solution

Afterwards, first we compare the performances of the heuristic methods and then analyze the working structure of the heuristic methods. For the performance comparison, initially we consider the optimality gaps of heuristic methods over 23 flight network. Then, we compare the improvement of the heuristic methods in comparison to the best solution generated by the integrated method 5400 sec. over 35 flight network. Also, the improvement of the heuristic methods in comparison to the published schedule over 114 flight network is presented. Eventually, we analyze the working structure of the heuristic methods. For heuristic1, the effect of CSCM and for heuristic2 the cost change after root node are analyzed over 23, 35 and 114 flight networks.

# Chapter 7

## Conclusions and Future Works

This final chapter is devoted to the summary of thesis and the future research directions. In the summary, we define our problem briefly and discuss the contributions of our study. Afterwards, we list the possible extensions and the future research directions.

### 7.1 Summary of Thesis

In this study, we solve robust airline schedule design, aircraft fleet and routing problems within a daily planning horizon for a given set of flights and a set of aircraft in an integrated manner while considering maintenance requirements and passengers' connection service levels.

For an integration of robust airline scheduling, aircraft fleet and routing problems, it is the first time when cruise speed/time is controlled. This novel consideration of cruise speed/time control enables us to make following contributions. First, aircraft utilization could be increased and even total number of aircraft needed to cover a set of flights could be decreased while ensuring equivalent service level and maintenance requirements. Moreover, due to this increase in the utilization of efficient aircraft, total cost of fuel consumption could be decreased. Second, the robustness can be achieved with smaller cost. To elaborate,

on a route having a flight with a great delay probability would require idle time insertion for the following flights to be performed on time; while removing the problematic flight from that sequence could render that intervention unnecessary. Due to cruise speed control, our study has more options to make this type of changes on routing decisions.

In addition to the contributions due to cruise speed control, we put forward the following contributions regarding the solution of the problem. First, we propose a nonlinear mixed integer programming model and present its second order conic reformulation. Furthermore, to solve large scale problems faster, we propose two heuristic methods which are discrete approximation and cruise speed control algorithm and multi-stage triplet search algorithm.

Eventually, we conduct a computational study in which we analyze these three proposed solution methods and compare their performances among each other and against the published schedule.

## 7.2 Future Works

Since we deal with the integration of three airline schedule planning problems, one immediate future research direction can be integration of the last problem, crew assignment problem. Although Papadakos [34] proposes an approach which integrates crew assignment problem to these three stages; as far as we know, there is not a study which integrates these four problems within the consideration of passengers' service levels and cruise speed control.

As another further research direction, the effect of the different parameters on the performances of the heuristic methods can be analyzed. In this study, after several trials, we choose the values and use those values as fixed parameters for our heuristic methods such as discretization interval in discrete approximation and cruise speed control algorithm or beam size and depth size in multi-stage triplet search algorithm. However, the performance of the heuristic methods can change by depending on those parameters.

One last direction would be a study in which cruise speed control decisions are given dynamically after the realizations of departure/arrival times on the day of operation. To be more precise, instead of following cruise speed/time instructions which are given prior to day of operation, deciding on cruise speed of a flight just after the departure can prevent the delay propagation immediately.



# Bibliography

- [1] A. Ben-Tal and A. Nemirovski, *Lectures on modern convex optimization: analysis, algorithms, and engineering applications*, vol. 2. Siam, 2001.
- [2] O. Günlük and J. Linderoth, “Perspective reformulations of mixed integer nonlinear programs with indicator variables,” *Mathematical programming*, vol. 124, no. 1-2, pp. 183–205, 2010.
- [3] C. Barnhart and A. Cohn, “Airline schedule planning: Accomplishments and opportunities,” *Manufacturing & Service Operations Management*, vol. 6, no. 1, pp. 3–22, 2004.
- [4] R. Gopalan and K. T. Talluri, “Mathematical models in airline schedule planning: A survey,” *Annals of Operations Research*, vol. 76, pp. 155–185, 1998.
- [5] J. Clausen, A. Larsen, J. Larsen, and N. J. Rezanova, “Disruption management in the airline industry concepts, models and methods,” *Computers & Operations Research*, vol. 37, no. 5, pp. 809–821, 2010.
- [6] M. Lapp, *Methods for Improving Robustness and Recovery in Aviation Planning*. PhD thesis, The University of Michigan, 2012.
- [7] S. Lan, *Planning for robust airline operations: Optimizing aircraft routings and flight departure times to achieve minimum passenger disruptions*. PhD thesis, Massachusetts Institute of Technology, 2003.

- [8] M. Arikan, V. Deshpande, and M. Sohoni, “Building reliable air-travel infrastructure using empirical data and stochastic models of airline networks,” *Operations Research*, vol. 61, no. 1, pp. 45–64, 2013.
- [9] S. Ahmadbeygi, A. Cohn, and M. Lapp, “Decreasing airline delay propagation by re-allocating scheduled slack,” *IIE transactions*, vol. 42, no. 7, pp. 478–489, 2010.
- [10] O. Weide, *Robust and integrated airline scheduling*. PhD thesis, University of Auckland, 2009.
- [11] C. Barnhart, P. Belobaba, and A. R. Odoni, “Applications of operations research in the air transport industry,” *Transportation science*, vol. 37, no. 4, pp. 368–391, 2003.
- [12] A. S. Duran, S. Gürel, and M. S. Aktürk, “Robust airline scheduling with controllable cruise times and chance constraints,” *to appear in IIE Transactions*, 2014.
- [13] M. M. Etschmaier and D. F. Mathaisel, “Airline scheduling: An overview,” *Transportation Science*, vol. 19, no. 2, pp. 127–138, 1985.
- [14] J. Abara, “Applying integer linear programming to the fleet assignment problem,” *Interfaces*, vol. 19, no. 4, pp. 20–28, 1989.
- [15] C. A. Hane, C. Barnhart, E. L. Johnson, R. E. Marsten, G. L. Nemhauser, and G. Sigismondi, “The fleet assignment problem: solving a large-scale integer program,” *Mathematical Programming*, vol. 70, no. 1-3, pp. 211–232, 1995.
- [16] T. L. Jacobs, B. C. Smith, and E. L. Johnson, “Incorporating network flow effects into the airline fleet assignment process,” *Transportation Science*, vol. 42, no. 4, pp. 514–529, 2008.
- [17] C. Barnhart, T. S. Kniker, and M. Lohatepanont, “Itinerary-based airline fleet assignment,” *Transportation Science*, vol. 36, no. 2, pp. 199–217, 2002.

- [18] B. C. Smith and E. L. Johnson, “Robust airline fleet assignment: Imposing station purity using station decomposition,” *Transportation Science*, vol. 40, no. 4, pp. 497–516, 2006.
- [19] H. D. Sherali, E. K. Bish, and X. Zhu, “Airline fleet assignment concepts, models, and algorithms,” *European Journal of Operational Research*, vol. 172, no. 1, pp. 1–30, 2006.
- [20] R. Gopalan and K. T. Talluri, “The aircraft maintenance routing problem,” *Operations Research*, vol. 46, no. 2, pp. 260–271, 1998.
- [21] L. Clarke, E. Johnson, G. Nemhauser, and Z. Zhu, “The aircraft rotation problem,” *Annals of Operations Research*, vol. 69, pp. 33–46, 1997.
- [22] C. Sriram and A. Haghani, “An optimization model for aircraft maintenance scheduling and re-assignment,” *Transportation Research Part A: Policy and Practice*, vol. 37, no. 1, pp. 29–48, 2003.
- [23] M. Haouari, S. Shao, and H. D. Sherali, “A lifted compact formulation for the daily aircraft maintenance routing problem,” *Transportation Science*, vol. 47, no. 4, pp. 508–525, 2012.
- [24] M. A. Aloulou, M. Haouari, and F. Z. Mansour, “A model for enhancing robustness of aircraft and passenger connections,” *Transportation Research Part C: Emerging Technologies*, vol. 32, no. 0, pp. 48 – 60, 2013.
- [25] B. Rexing, C. Barnhart, T. Kniker, A. Jarrah, and N. Krishnamurthy, “Airline fleet assignment with time windows,” *Transportation Science*, vol. 34, no. 1, pp. 1–20, 2000.
- [26] M. Lohatepanont and C. Barnhart, “Airline schedule planning: Integrated models and algorithms for schedule design and fleet assignment,” *Transportation Science*, vol. 38, no. 1, pp. 19–32, 2004.
- [27] H. D. Sherali, K.-H. Bae, and M. Haouari, “A benders decomposition approach for an integrated airline schedule design and fleet assignment problem with flight retiming, schedule balance, and demand recapture,” *Annals of Operations Research*, vol. 210, no. 1, pp. 213–244, 2013.

- [28] O. Şafak, “Fleet type assignment and robust airline scheduling with chance constraints under environmental emission considerations,” Master’s thesis, Bilkent University, 2013.
- [29] C. Barnhart, N. L. Boland, L. W. Clarke, E. L. Johnson, G. L. Nemhauser, and R. G. Shenoi, “Flight string models for aircraft fleet assignment and routing,” *Transportation Science*, vol. 32, no. 3, pp. 208–220, 1998.
- [30] M. Grönkvist, *The tail assignment problem*. PhD thesis, Chalmers University of Technology, 2005.
- [31] Z. Liang and W. A. Chaovalitwongse, “A network-based model for the integrated weekly aircraft maintenance routing and fleet assignment problem,” *Transportation Science*, vol. 47, no. 4, pp. 493–507, 2012.
- [32] G. Desaulniers, J. Desrosiers, Y. Dumas, M. M. Solomon, and F. Soumis, “Daily aircraft routing and scheduling,” *Management Science*, vol. 43, no. 6, pp. 841–855, 1997.
- [33] H. D. Sherali, K.-H. Bae, and M. Haouari, “An integrated approach for airline flight selection and timing, fleet assignment, and aircraft routing,” *Transportation Science*, vol. 47, no. 4, pp. 455–476, 2013.
- [34] N. Papadacos, “Integrated airline scheduling,” *Computers & Operations Research*, vol. 36, no. 1, pp. 176–195, 2009.
- [35] M. S. Aktürk, A. Atamtürk, and S. Gürel, “A strong conic quadratic reformulation for machine-job assignment with controllable processing times,” *Operations Research Letters*, vol. 37, no. 3, pp. 187–191, 2009.
- [36] IATA, “Airline fuel and labor cost share.” [http://www.iata.org/whatwedo/Documents/economics/Airline\\_Labour\\_Cost\\_Share\\_Feb2010.pdf](http://www.iata.org/whatwedo/Documents/economics/Airline_Labour_Cost_Share_Feb2010.pdf), 2010. Visited June 2014.
- [37] Airbus, “Airbus flight operations support and line assistance, getting to grips with the cost index.” [http://www.iata.org/whatwedo/Documents/fuel/airbus\\_cost\\_index\\_material.pdf](http://www.iata.org/whatwedo/Documents/fuel/airbus_cost_index_material.pdf), 1998. Visited November 2012.

- [38] M. S. Aktürk, A. Atamtürk, and S. Gürel, “Aircraft rescheduling with cruise speed control,” *to appear in Operations Research*, 2014.
- [39] U. Arıkan, S. Gürel, and M. S. Aktürk, “Integrated aircraft and passenger recovery with cruise time controllability,” *to appear in Annals of Operations Research*, 2013.
- [40] EUROCONTROL, “User manual for the base of aircraft data (bada) revision 3.10.,” Tech. Rep. 12/04/10-45, EUROCONTROL Experimental Centre Centre de Bois des Bordes B.P.15 F - 91222 Brtigny-sur-Orge CEDEX FRANCE, 2012.
- [41] A. Parmentier, “Aircraft routing: complexity and algorithms,” Master’s thesis, Ecole des Ponts ParisTech, 2013.
- [42] J.-B. Hiriart-Urruty and C. Lemaréchal, *Fundamentals of convex analysis*. Springer, 2001.
- [43] BTS, “Airline on-time performance data.” [http://www.transtats.bts.gov/DL\\_SelectFields.asp?Table\\_ID=236&DB\\_Short\\_Name=On-Time.](http://www.transtats.bts.gov/DL_SelectFields.asp?Table_ID=236&DB_Short_Name=On-Time.), 2010. Visited June 2012.
- [44] L. Delgado and X. Prats, “Fuel consumption assessment for speed variation concepts during the cruise phase,” in *Proceedings of the Conference on Air Traffic Management (ATM) Economics*, 2009.

# Appendix A

## Computational Results

### A.1 23 Flight Network

Table A.1: Cost for the schedule generated by the integrated model

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	107208.4	2795.4	61200.0	2706.8	173910.6
2	0	0	0	1	1	107208.4	3117.2	61200.0	2706.8	174232.4
3	0	0	0	2	1	107208.4	3848.3	61200.0	2706.8	174963.5
4	0	0	2	0	1	120187.0	2254.2	522000.0	2022.1	646463.3
5	0	0	2	1	1	120187.0	2464.3	522000.0	2022.1	646673.5
6	0	0	2	2	1	120187.0	2941.8	522000.0	2022.1	647151.0
7	0	2	0	0	1	108692.2	2209.1	61200.0	8542.2	180643.5
8	0	2	0	1	1	108692.2	2524.1	61200.0	8542.2	180958.5
9	0	2	0	2	1	108692.2	3239.7	61200.0	8542.2	181674.1
10	0	2	2	0	1	120187.1	2254.2	522000.0	8088.5	652529.7
11	0	2	2	1	1	120187.0	2464.3	522000.0	8088.5	652739.8
12	0	2	2	2	1	120187.4	2942.6	522000.0	8088.5	653218.4
13	2	0	0	0	1	214416.7	2795.4	61200.0	2706.8	281119.0
14	2	0	0	1	1	214416.7	3117.2	61200.0	2706.8	281440.8
15	2	0	0	2	1	214416.7	3848.5	61200.0	2706.8	282172.0
16	2	0	2	0	1	240374.1	2254.2	522000.0	2022.1	766650.4
17	2	0	2	1	1	240374.1	2464.6	522000.0	2022.1	766860.8
18	2	0	2	2	1	240374.1	2941.7	522000.0	2022.1	767337.9
19	2	2	0	0	1	214416.7	2795.4	61200.0	10827.4	289239.5
20	2	2	0	1	1	214416.7	3117.2	61200.0	10827.4	289561.3
21	2	2	0	2	1	214416.7	3848.3	61200.0	10827.4	290292.4
22	2	2	2	0	1	240374.1	2254.2	522000.0	8088.5	772716.7
23	2	2	2	1	1	240374.1	2464.3	522000.0	8088.5	772926.9

*Continued on next page*

Table A.1 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
24	2	2	2	2	1	240374.1	2941.8	522000.0	8088.5	773404.4
1	0	0	0	0	2	107210.3	1755.4	61200.0	1594.7	171760.4
2	0	0	0	1	2	107209.6	2077.2	61200.0	1594.7	172081.5
3	0	0	0	2	2	107208.5	2808.3	61200.0	1594.7	172811.5
4	0	0	2	0	2	120188.4	1676.2	522000.0	1368.6	645233.2
5	0	0	2	1	2	120187.9	1886.3	522000.0	1368.6	645442.8
6	0	0	2	2	2	120187.1	2363.7	522000.0	1368.6	645919.5
7	0	2	0	0	2	108692.5	1439.1	61200.0	3826.4	175157.9
8	0	2	0	1	2	108692.5	1658.3	61200.0	3826.4	175377.2
9	0	2	0	2	2	108692.3	2249.7	61200.0	3826.4	175968.4
10	0	2	2	0	2	120188.4	1676.3	522000.0	5474.5	649339.3
11	0	2	2	1	2	120187.9	1886.6	522000.0	5474.5	649549.0
12	0	2	2	2	2	120187.2	2363.8	522000.0	5474.5	650025.5
13	2	0	0	0	2	214420.7	1755.4	61200.0	1594.7	278970.8
14	2	0	0	1	2	214419.1	2077.2	61200.0	1594.7	279291.0
15	2	0	0	2	2	214417.0	2808.3	61200.0	1594.7	280020.0
16	2	0	2	0	2	240376.9	1676.2	522000.0	1368.6	765421.7
17	2	0	2	1	2	240375.8	1886.3	522000.0	1368.6	765630.7
18	2	0	2	2	2	240374.3	2363.7	522000.0	1368.6	766106.6
19	2	2	0	0	2	214420.7	1755.6	61200.0	6378.7	283755.0
20	2	2	0	1	2	217384.9	1658.3	61200.0	3826.4	284069.7
21	2	2	0	2	2	217384.5	2249.7	61200.0	3826.4	284660.7
22	2	2	2	0	2	240376.9	1676.2	522000.0	5474.5	769527.5
23	2	2	2	1	2	240375.8	1886.3	522000.0	5474.5	769736.6
24	2	2	2	2	2	240374.3	2363.7	522000.0	5474.5	770212.5
1	0	0	0	0	3	107208.7	1917.4	61200.0	5358.6	175684.7
2	0	0	0	1	3	107208.4	2239.2	61200.0	5358.6	176006.2
3	0	0	0	2	3	107208.4	2970.3	61200.0	5358.6	176737.3
4	0	0	2	0	3	120187.2	1538.2	522000.0	3438.4	647163.8
5	0	0	2	1	3	120187.1	1748.3	522000.0	3438.4	647373.8
6	0	0	2	2	3	120187.0	2225.7	522000.0	3438.4	647851.1
7	0	2	0	0	3	120187.2	1538.2	52200.0	13753.7	187679.1
8	0	2	0	1	3	120187.1	1748.3	52200.0	13753.7	187889.1
9	0	2	0	2	3	120187.0	2225.7	52200.0	13753.7	188366.4
10	0	2	2	0	3	120187.2	1538.2	522000.0	13753.7	657479.1
11	0	2	2	1	3	120187.1	1748.3	522000.0	13753.7	657689.1
12	0	2	2	2	3	120187.0	2225.7	522000.0	13753.7	658166.4
13	2	0	0	0	3	214417.2	1917.4	61200.0	5358.6	282893.3
14	2	0	0	1	3	214416.8	2239.2	61200.0	5358.6	283214.6
15	2	0	0	2	3	214416.7	2970.3	61200.0	5358.6	283945.7
16	2	0	2	0	3	240374.4	1538.2	522000.0	3438.4	767351.0
17	2	0	2	1	3	240374.1	1748.3	522000.0	3438.4	767560.9
18	2	0	2	2	3	240374.1	2225.8	522000.0	3438.4	768038.3
19	2	2	0	0	3	214417.2	1917.4	61200.0	21434.5	298969.1
20	2	2	0	1	3	214416.8	2239.2	61200.0	21434.5	299290.5
21	2	2	0	2	3	214416.7	2970.3	61200.0	21434.5	300021.5
22	2	2	2	0	3	240374.4	1538.2	522000.0	13753.7	777666.3

Continued on next page

Table A.1 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
23	2	2	2	1	3	240374.1	1748.6	522000.0	13753.7	777876.4
24	2	2	2	2	3	240374.1	2225.7	522000.0	13753.7	778353.5

Table A.2: Cost for the schedule generated by heuristic1

Run	Factors				Rep.	Costs					Before CSCM	
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total	Fuel	Idle Time
1	0	0	0	0	1	107208.4	2795.4	61200.0	2706.8	173910.7	107261.7	2795.4
2	0	0	0	1	1	107208.4	3117.2	61200.0	2706.8	174232.4	107261.7	3117.2
3	0	0	0	2	1	107208.4	3848.3	61200.0	2706.8	174963.6	107261.7	3848.3
4	0	0	2	0	1	120187.1	2254.3	522000.0	2022.1	646463.5	120308.1	2254.2
5	0	0	2	1	1	120187.0	2464.3	522000.0	2022.1	646673.5	120308.1	2464.3
6	0	0	2	2	1	120187.1	2941.8	522000.0	2022.1	647151.1	120308.1	2941.7
7	0	2	0	0	1	108692.2	2209.1	61200.0	8542.2	180643.5	108736.4	2209.1
8	0	2	0	1	1	108692.4	2524.4	61200.0	8542.2	180959.0	108736.4	2524.1
9	0	2	0	2	1	108692.2	3239.8	61200.0	8542.2	181674.2	108736.4	3239.7
10	0	2	2	0	1	120187.1	2254.3	522000.0	8088.5	652529.8	120308.1	2254.2
11	0	2	2	1	1	120187.0	2464.3	522000.0	8088.5	652739.9	120308.1	2464.3
12	0	2	2	2	1	120187.1	2941.8	522000.0	8088.5	653217.4	120308.1	2941.7
13	2	0	0	0	1	214416.7	2795.5	61200.0	2706.8	281119.0	214523.4	2795.4
14	2	0	0	1	1	214416.7	3117.2	61200.0	2706.8	281440.8	214523.4	3117.2
15	2	0	0	2	1	214419.2	3850.9	61200.0	2706.8	282176.9	214523.4	3848.3
16	2	0	2	0	1	240374.1	2254.2	522000.0	2022.1	766650.3	240616.1	2254.2
17	2	0	2	1	1	240374.1	2464.3	522000.0	2022.1	766860.5	240616.1	2464.3
18	2	0	2	2	1	240374.1	2941.7	522000.0	2022.1	767337.9	240616.1	2941.7
19	2	2	0	0	1	214416.7	2795.5	61200.0	10827.4	289239.6	214523.4	2795.4
20	2	2	0	1	1	214416.7	3117.2	61200.0	10827.4	289561.3	214523.4	3117.2
21	2	2	0	2	1	214419.2	3850.9	61200.0	10827.4	290297.4	214523.4	3848.3
22	2	2	2	0	1	240374.1	2254.2	522000.0	8088.5	772716.7	240616.1	2254.2
23	2	2	2	1	1	240374.1	2464.3	522000.0	8088.5	772926.9	240616.1	2464.3
24	2	2	2	2	1	240374.1	2941.7	522000.0	8088.5	773404.2	240616.1	2941.7
1	0	0	0	0	2	107210.3	1755.4	61200.0	1594.7	171760.4	107298.2	1755.4
2	0	0	0	1	2	107209.6	2077.4	61200.0	1594.7	172081.7	107298.2	2077.2
3	0	0	0	2	2	107208.5	2808.3	61200.0	1594.7	172811.5	107298.2	2808.3
4	0	0	2	0	2	120188.4	1676.2	522000.0	1368.6	645233.3	120334.1	1676.2
5	0	0	2	1	2	120187.9	1886.3	522000.0	1368.6	645442.8	120334.1	1886.3
6	0	0	2	2	2	120187.2	2363.7	522000.0	1368.6	645919.5	120341.9	2363.7
7	0	2	0	0	2	108692.5	1439.1	61200.0	3826.4	175158.0	108758.6	1439.1
8	0	2	0	1	2	108692.5	1658.4	61200.0	3826.4	175377.3	108758.6	1658.3
9	0	2	0	2	2	108692.3	2249.8	61200.0	3826.4	175968.4	108758.6	2249.7
10	0	2	2	0	2	120188.4	1676.2	522000.0	5474.5	649339.2	120334.1	1676.2
11	0	2	2	1	2	120187.9	1886.3	522000.0	5474.5	649548.7	120334.1	1886.3
12	0	2	2	2	2	120187.2	2363.7	522000.0	5474.5	650025.4	120334.1	2363.7
13	2	0	0	0	2	214420.7	1755.4	61200.0	1594.7	278970.8	214596.4	1755.4
14	2	0	0	1	2	214419.2	2077.4	61200.0	1594.7	279291.3	214596.4	2077.2
15	2	0	0	2	2	214417.1	2808.4	61200.0	1594.7	280020.1	214596.4	2808.3
16	2	0	2	0	2	240377.1	1676.6	522000.0	1368.6	765422.3	240668.2	1676.2
17	2	0	2	1	2	240375.8	1886.3	522000.0	1368.6	765630.7	240668.2	1886.3
18	2	0	2	2	2	240374.3	2363.7	522000.0	1368.6	766106.6	240668.2	2363.7
19	2	2	0	0	2	214420.7	1755.4	61200.0	6378.7	283754.9	214596.4	1755.4
20	2	2	0	1	2	217384.9	1658.3	61200.0	3826.4	284069.7	217517.2	1658.3
21	2	2	0	2	2	217384.5	2249.7	61200.0	3826.4	284660.7	217517.2	2249.7
22	2	2	2	0	2	240377.1	1676.6	522000.0	5474.5	769528.2	240668.2	1676.2
23	2	2	2	1	2	240375.8	1886.3	522000.0	5474.5	769736.6	240668.2	1886.3
24	2	2	2	2	2	240374.3	2363.7	522000.0	5474.5	770212.5	240668.2	2363.7
1	0	0	0	0	3	107209.4	1918.6	61200.0	5358.6	175686.6	107298.2	1917.4
2	0	0	0	1	3	107208.4	2239.2	61200.0	5358.6	176006.3	107298.2	2239.2

Continued on next page



Table A.2 – Continued from previous page

Run	Factors				Rep.	Costs					Before CSCM	
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total	Fuel	Idle Time
3	0	0	0	2	3	107208.4	2970.3	61200.0	5358.6	176737.3	107261.7	2970.3
4	0	0	2	0	3	120187.2	1538.2	522000.0	3438.4	647163.8	120334.1	1538.2
5	0	0	2	1	3	120187.1	1748.3	522000.0	3438.4	647373.8	120334.1	1748.3
6	0	0	2	2	3	120187.0	2225.7	522000.0	3438.4	647851.1	120308.1	2225.7
7	0	2	0	0	3	120187.2	1538.2	52200.0	13753.7	187679.1	120334.1	1538.2
8	0	2	0	1	3	120187.1	1748.3	52200.0	13753.7	187889.1	120334.1	1748.3
9	0	2	0	2	3	120187.0	2225.7	52200.0	13753.7	188366.4	120308.1	2225.7
10	0	2	2	0	3	120187.2	1538.2	522000.0	13753.7	657479.1	120334.1	1538.2
11	0	2	2	1	3	120187.1	1748.3	522000.0	13753.7	657689.1	120334.1	1748.3
12	0	2	2	2	3	120187.0	2225.7	522000.0	13753.7	658166.4	120308.1	2225.7
13	2	0	0	0	3	214417.2	1917.4	61200.0	5358.6	282893.3	214596.4	1917.4
14	2	0	0	1	3	214416.8	2239.3	61200.0	5358.6	283214.8	214523.4	2302.7
15	2	0	0	2	3	214416.7	2970.4	61200.0	5358.6	283945.7	214523.4	2970.3
16	2	0	2	0	3	240374.4	1538.2	522000.0	3438.4	767351.0	240668.2	1538.2
17	2	0	2	1	3	240374.1	1748.3	522000.0	3438.4	767560.9	240668.2	1748.3
18	2	0	2	2	3	240374.1	2225.7	522000.0	3438.4	768038.2	240616.1	2225.7
19	2	2	0	0	3	214417.2	1917.4	61200.0	21434.5	298969.1	214596.4	1917.4
20	2	2	0	1	3	214416.8	2239.3	61200.0	21434.5	299290.6	214523.4	2302.7
21	2	2	0	2	3	214416.7	2970.4	61200.0	21434.5	300021.6	214523.4	2970.3
22	2	2	2	0	3	240374.4	1538.2	522000.0	13753.7	777666.3	240668.2	1538.2
23	2	2	2	1	3	240374.1	1748.3	522000.0	13753.7	777876.2	240668.2	1748.3
24	2	2	2	2	3	240374.1	2225.7	522000.0	13753.7	778353.5	240616.1	2225.7

Table A.3: Cost for the schedule generated by heuristic2

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	108692.3	2209.4	61200.0	2135.6	174237.3
2	0	0	0	1	1	108692.3	2524.4	61200.0	2135.6	174552.2
3	0	0	0	2	1	108941.1	3240.7	61200.0	2135.6	175517.4
4	0	0	2	0	1	120187.5	2254.5	522000.0	2022.1	646464.0
5	0	0	2	1	1	120187.4	2464.6	522000.0	2022.1	646674.2
6	0	0	2	2	1	120187.5	2942.0	522000.0	2022.1	647151.5
7	0	2	0	0	1	108692.3	2209.4	61200.0	8542.2	180644.0
8	0	2	0	1	1	108942.2	2525.1	61200.0	8542.2	181209.5
9	0	2	0	2	1	108692.3	3240.0	61200.0	8542.2	181674.5
10	0	2	2	0	1	120187.5	2254.5	522000.0	8088.5	652530.4
11	0	2	2	1	1	120187.4	2464.6	522000.0	8088.5	652740.5
12	0	2	2	2	1	120187.5	2942.0	522000.0	8088.5	653217.9
13	2	0	0	0	1	217383.7	2209.6	61200.0	2135.6	282928.9
14	2	0	0	1	1	217383.7	2524.5	61200.0	2135.6	283243.8
15	2	0	0	2	1	217382.3	3241.3	61200.0	2135.6	283959.2
16	2	0	2	0	1	240373.8	2255.4	522000.0	2022.1	766651.3
17	2	0	2	1	1	240373.9	2465.3	522000.0	2022.1	766861.3
18	2	0	2	2	1	240373.9	2942.4	522000.0	2022.1	767338.5
19	2	2	0	0	1	217383.7	2209.6	61200.0	8542.2	289335.5
20	2	2	0	1	1	217383.7	2524.5	61200.0	8542.2	289650.4
21	2	2	0	2	1	217382.3	3241.3	61200.0	8542.2	290365.8
22	2	2	2	0	1	240373.8	2255.4	522000.0	8088.5	772717.7
23	2	2	2	1	1	240373.9	2465.3	522000.0	8088.5	772927.7

Continued on next page

Table A.3 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
24	2	2	2	2	1	240373.9	2942.4	522000.0	8088.5	773404.8
1	0	0	0	0	2	120188.8	1676.4	52200.0	1368.6	175433.8
2	0	0	0	1	2	107209.8	2077.7	61200.0	1594.7	172082.3
3	0	0	0	2	2	108693.5	2251.5	61200.0	956.6	173101.6
4	0	0	2	0	2	120188.8	1676.4	522000.0	1368.6	645233.8
5	0	0	2	1	2	120189.5	1887.3	522000.0	1368.6	645445.3
6	0	0	2	2	2	120187.9	2364.1	522000.0	1368.6	645920.7
7	0	2	0	0	2	108693.3	1440.2	61200.0	3826.4	175159.9
8	0	2	0	1	2	108693.6	1659.9	61200.0	3826.4	175380.0
9	0	2	0	2	2	108693.5	2251.5	61200.0	3826.4	175971.4
10	0	2	2	0	2	120188.8	1676.4	522000.0	5474.5	649339.7
11	0	2	2	1	2	120189.5	1887.3	522000.0	5474.5	649551.2
12	0	2	2	2	2	120187.9	2364.1	522000.0	5474.5	650026.6
13	2	0	0	0	2	223579.2	1375.7	61200.0	3222.9	289377.8
14	2	0	0	1	2	223555.3	1700.0	61200.0	3222.9	289678.1
15	2	0	0	2	2	217382.5	2251.6	61200.0	956.6	281790.7
16	2	0	2	0	2	240376.6	1678.4	522000.0	1368.6	765423.6
17	2	0	2	1	2	240375.6	1888.4	522000.0	1368.6	765632.6
18	2	0	2	2	2	240374.1	2365.3	522000.0	1368.6	766108.1
19	2	2	0	0	2	217382.7	1441.2	61200.0	3826.4	283850.3
20	2	2	0	1	2	217382.7	1660.4	61200.0	3826.4	284069.5
21	2	2	0	2	2	217382.5	2251.6	61200.0	3826.4	284660.5
22	2	2	2	0	2	240376.6	1678.4	522000.0	5474.5	769529.4
23	2	2	2	1	2	240375.6	1888.4	522000.0	5474.5	769738.5
24	2	2	2	2	2	240374.1	2365.3	522000.0	5474.5	770213.9
1	0	0	0	0	3	108692.7	1361.8	61200.0	5006.0	176260.5
2	0	0	0	1	3	108692.7	1677.2	61200.0	5006.0	176575.8
3	0	0	0	2	3	108692.3	2392.1	61200.0	5006.0	177290.4
4	0	0	2	0	3	120188.8	1539.1	522000.0	3438.4	647166.4
5	0	0	2	1	3	120187.5	1748.6	522000.0	3438.4	647374.5
6	0	0	2	2	3	120187.5	2226.0	522000.0	3438.4	647851.9
7	0	2	0	0	3	108692.7	1361.8	61200.0	20023.9	191278.5
8	0	2	0	1	3	108692.7	1677.2	61200.0	20023.9	191593.8
9	0	2	0	2	3	108692.3	2392.1	61200.0	20023.9	192308.3
10	0	2	2	0	3	120188.8	1539.1	522000.0	13753.7	657481.7
11	0	2	2	1	3	120187.5	1748.6	522000.0	13753.7	657689.8
12	0	2	2	2	3	120187.5	2226.0	522000.0	13753.7	658167.2
13	2	0	0	0	3	217382.4	1363.2	61200.0	5006.0	284951.6
14	2	0	0	1	3	217382.5	1677.7	61200.0	5006.0	285266.2
15	2	0	0	2	3	217382.2	2393.4	61200.0	5006.0	285981.6
16	2	0	2	0	3	240374.4	1538.6	522000.0	3438.4	767351.4
17	2	0	2	1	3	240379.4	1748.7	522000.0	3438.4	767566.5
18	2	0	2	2	3	240373.7	2227.4	522000.0	3438.4	768039.6
19	2	2	0	0	3	217382.4	1363.2	61200.0	20023.9	299969.6
20	2	2	0	1	3	217382.5	1677.7	61200.0	20023.9	300284.2
21	2	2	0	2	3	217382.2	2393.4	61200.0	20023.9	300999.5
22	2	2	2	0	3	240374.4	1538.6	522000.0	13753.7	777666.7

Continued on next page

Table A.3 – *Continued from previous page*

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
23	2	2	2	1	3	240379.4	1748.7	522000.0	13753.7	777881.8
24	2	2	2	2	3	240373.7	2227.4	522000.0	13753.7	778354.9

Table A.4: Cost for the schedule generated by heuristic2 at the root node

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	128113.5	2255.5	52800.0	1356.3	184525.3
2	0	0	0	1	1	128113.9	2466.3	52800.0	1356.3	184736.4
3	0	0	0	2	1	128113.0	2942.5	52800.0	1356.3	185211.8
4	0	0	2	0	1	120187.5	2254.5	522000.0	2022.1	646464.0
5	0	0	2	1	1	120187.4	2464.6	522000.0	2022.1	646674.2
6	0	0	2	2	1	120187.5	2942.0	522000.0	2022.1	647151.5
7	0	2	0	0	1	128113.5	2255.5	52800.0	5425.2	188594.2
8	0	2	0	1	1	128113.9	2466.3	52800.0	5425.2	188805.3
9	0	2	0	2	1	128113.0	2942.5	52800.0	5425.2	189280.7
10	0	2	2	0	1	120187.5	2254.5	522000.0	8088.5	652530.4
11	0	2	2	1	1	120187.4	2464.6	522000.0	8088.5	652740.5
12	0	2	2	2	1	120187.5	2942.0	522000.0	8088.5	653217.9
13	2	0	0	0	1	256223.9	2254.6	52800.0	1356.3	312634.8
14	2	0	0	1	1	256222.9	2466.3	52800.0	1356.3	312845.5
15	2	0	0	2	1	256224.0	2942.1	52800.0	1356.3	313322.4
16	2	0	2	0	1	240373.8	2255.4	522000.0	2022.1	766651.3
17	2	0	2	1	1	240373.9	2465.3	522000.0	2022.1	766861.3
18	2	0	2	2	1	240373.9	2942.4	522000.0	2022.1	767338.5
19	2	2	0	0	1	256223.9	2254.6	52800.0	5425.2	316703.7
20	2	2	0	1	1	257214.8	2464.8	52800.0	5425.2	317904.7
21	2	2	0	2	1	257214.9	2942.0	52800.0	5425.2	318382.1
22	2	2	2	0	1	241364.8	2254.5	522000.0	8088.5	773707.8
23	2	2	2	1	1	241364.8	2464.6	522000.0	8088.5	773917.9
24	2	2	2	2	1	240373.9	2942.4	522000.0	8088.5	773404.8
1	0	0	0	0	2	129107.9	1439.8	52800.0	956.6	184304.2
2	0	0	0	1	2	129107.9	1659.0	52800.0	956.6	184523.5
3	0	0	0	2	2	128113.9	2365.2	52800.0	730.6	184009.7
4	0	0	2	0	2	121803.8	1439.3	522000.0	1594.7	646837.8
5	0	0	2	1	2	121183.5	1659.3	522000.0	1594.7	646437.5
6	0	0	2	2	2	120187.9	2364.1	522000.0	1368.6	645920.7
7	0	2	0	0	2	128123.3	1676.3	52800.0	2922.2	185521.8
8	0	2	0	1	2	128122.4	1886.5	52800.0	2922.2	185731.1
9	0	2	0	2	2	128113.9	2365.2	52800.0	2922.2	186201.3
10	0	2	2	0	2	120188.8	1676.4	522000.0	5474.5	649339.7
11	0	2	2	1	2	120189.5	1887.3	522000.0	5474.5	649551.2
12	0	2	2	2	2	120187.9	2364.1	522000.0	5474.5	650026.6

*Continued on next page*

Table A.4 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
13	2	0	0	0	2	258211.9	1442.2	52800.0	956.6	313410.7
14	2	0	0	1	2	258212.1	1661.3	52800.0	956.6	313630.1
15	2	0	0	2	2	256223.9	2364.6	52800.0	730.6	312119.1
16	2	0	2	0	2	242364.2	1440.0	522000.0	1594.7	767398.8
17	2	0	2	1	2	242363.7	1659.9	522000.0	1594.7	767618.4
18	2	0	2	2	2	240374.1	2365.3	522000.0	1368.6	766108.1
19	2	2	0	0	2	256226.9	1676.5	52800.0	2922.2	313625.6
20	2	2	0	1	2	256225.7	1886.7	52800.0	2922.2	313834.7
21	2	2	0	2	2	256223.9	2364.6	52800.0	2922.2	314310.7
22	2	2	2	0	2	240376.6	1678.4	522000.0	5474.5	769529.4
23	2	2	2	1	2	240375.6	1888.4	522000.0	5474.5	769738.5
24	2	2	2	2	2	240374.1	2365.3	522000.0	5474.5	770213.9
1	0	0	0	0	3	128114.2	1540.0	52800.0	2745.5	185199.7
2	0	0	0	1	3	128129.2	1748.9	52800.0	2745.5	185423.5
3	0	0	0	2	3	128112.4	2226.0	52800.0	2745.5	185883.8
4	0	0	2	0	3	120188.8	1539.1	522000.0	3438.4	647166.4
5	0	0	2	1	3	120187.5	1748.6	522000.0	3438.4	647374.5
6	0	0	2	2	3	120187.5	2226.0	522000.0	3438.4	647851.9
7	0	2	0	0	3	128114.2	1540.0	52800.0	10981.9	193436.1
8	0	2	0	1	3	128129.2	1748.9	52800.0	10981.9	193660.0
9	0	2	0	2	3	128112.4	2226.0	52800.0	10981.9	194120.3
10	0	2	2	0	3	120188.8	1539.1	522000.0	13753.7	657481.7
11	0	2	2	1	3	120187.5	1748.6	522000.0	13753.7	657689.8
12	0	2	2	2	3	120187.5	2226.0	522000.0	13753.7	658167.2
13	2	0	0	0	3	256224.2	1538.8	52800.0	2745.5	313308.5
14	2	0	0	1	3	256223.8	1749.1	52800.0	2745.5	313518.4
15	2	0	0	2	3	256222.6	2227.8	52800.0	2745.5	313995.9
16	2	0	2	0	3	240374.4	1538.6	522000.0	3438.4	767351.4
17	2	0	2	1	3	240379.4	1748.7	522000.0	3438.4	767566.5
18	2	0	2	2	3	240373.7	2227.4	522000.0	3438.4	768039.6
19	2	2	0	0	3	256224.2	1538.8	52800.0	10981.9	321544.9
20	2	2	0	1	3	256223.8	1749.1	52800.0	10981.9	321754.8
21	2	2	0	2	3	256222.6	2227.8	52800.0	10981.9	322232.3
22	2	2	2	0	3	240374.4	1538.6	522000.0	13753.7	777666.7
23	2	2	2	1	3	240379.4	1748.7	522000.0	13753.7	777881.8
24	2	2	2	2	3	240373.7	2227.4	522000.0	13753.7	778354.9

Table A.5: Cost for the published schedule

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	125403.9	35800.3	61200.0	0	222404.2
2	0	0	0	1	1	125403.9	35800.3	61200.0	0	222404.2

Continued on next page

Table A.5 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
3	0	0	0	2	1	125403.9	35800.3	61200.0	0	222404.2
4	0	0	2	0	1	125403.9	35800.3	612000.0	0	773204.2
5	0	0	2	1	1	125403.9	35800.3	612000.0	0	773204.2
6	0	0	2	2	1	125403.9	35800.3	612000.0	0	773204.2
7	0	2	0	0	1	125403.9	35800.3	61200.0	0	222404.2
8	0	2	0	1	1	125403.9	35800.3	61200.0	0	222404.2
9	0	2	0	2	1	125403.9	35800.3	61200.0	0	222404.2
10	0	2	2	0	1	125403.9	35800.3	612000.0	0	773204.2
11	0	2	2	1	1	125403.9	35800.3	612000.0	0	773204.2
12	0	2	2	2	1	125403.9	35800.3	612000.0	0	773204.2
13	2	0	0	0	1	250807.8	35800.3	61200.0	0	347808.1
14	2	0	0	1	1	250807.8	35800.3	61200.0	0	347808.1
15	2	0	0	2	1	250807.8	35800.3	61200.0	0	347808.1
16	2	0	2	0	1	250807.8	35800.3	612000.0	0	898608.1
17	2	0	2	1	1	250807.8	35800.3	612000.0	0	898608.1
18	2	0	2	2	1	250807.8	35800.3	612000.0	0	898608.1
19	2	2	0	0	1	250807.8	35800.3	61200.0	0	347808.1
20	2	2	0	1	1	250807.8	35800.3	61200.0	0	347808.1
21	2	2	0	2	1	250807.8	35800.3	61200.0	0	347808.1
22	2	2	2	0	1	250807.8	35800.3	612000.0	0	898608.1
23	2	2	2	1	1	250807.8	35800.3	612000.0	0	898608.1
24	2	2	2	2	1	250807.8	35800.3	612000.0	0	898608.1
1	0	0	0	0	2	125403.9	35800.3	61200.0	0	222404.2
2	0	0	0	1	2	125403.9	35800.3	61200.0	0	222404.2
3	0	0	0	2	2	125403.9	35800.3	61200.0	0	222404.2
4	0	0	2	0	2	125403.9	35800.3	612000.0	0	773204.2
5	0	0	2	1	2	125403.9	35800.3	612000.0	0	773204.2
6	0	0	2	2	2	125403.9	35800.3	612000.0	0	773204.2
7	0	2	0	0	2	125403.9	35800.3	61200.0	0	222404.2
8	0	2	0	1	2	125403.9	35800.3	61200.0	0	222404.2
9	0	2	0	2	2	125403.9	35800.3	61200.0	0	222404.2
10	0	2	2	0	2	125403.9	35800.3	612000.0	0	773204.2
11	0	2	2	1	2	125403.9	35800.3	612000.0	0	773204.2
12	0	2	2	2	2	125403.9	35800.3	612000.0	0	773204.2
13	2	0	0	0	2	250807.8	35800.3	61200.0	0	347808.1
14	2	0	0	1	2	250807.8	35800.3	61200.0	0	347808.1
15	2	0	0	2	2	250807.8	35800.3	61200.0	0	347808.1
16	2	0	2	0	2	250807.8	35800.3	612000.0	0	898608.1
17	2	0	2	1	2	250807.8	35800.3	612000.0	0	898608.1
18	2	0	2	2	2	250807.8	35800.3	612000.0	0	898608.1
19	2	2	0	0	2	250807.8	35800.3	61200.0	0	347808.1
20	2	2	0	1	2	250807.8	35800.3	61200.0	0	347808.1
21	2	2	0	2	2	250807.8	35800.3	61200.0	0	347808.1
22	2	2	2	0	2	250807.8	35800.3	612000.0	0	898608.1
23	2	2	2	1	2	250807.8	35800.3	612000.0	0	898608.1
24	2	2	2	2	2	250807.8	35800.3	612000.0	0	898608.1
1	0	0	0	0	3	125403.9	35800.3	61200.0	0	222404.2

Continued on next page

Table A.5 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
2	0	0	0	1	3	125403.9	35800.3	61200.0	0	222404.2
3	0	0	0	2	3	125403.9	35800.3	61200.0	0	222404.2
4	0	0	2	0	3	125403.9	35800.3	612000.0	0	773204.2
5	0	0	2	1	3	125403.9	35800.3	612000.0	0	773204.2
6	0	0	2	2	3	125403.9	35800.3	612000.0	0	773204.2
7	0	2	0	0	3	125403.9	35800.3	61200.0	0	222404.2
8	0	2	0	1	3	125403.9	35800.3	61200.0	0	222404.2
9	0	2	0	2	3	125403.9	35800.3	61200.0	0	222404.2
10	0	2	2	0	3	125403.9	35800.3	612000.0	0	773204.2
11	0	2	2	1	3	125403.9	35800.3	612000.0	0	773204.2
12	0	2	2	2	3	125403.9	35800.3	612000.0	0	773204.2
13	2	0	0	0	3	250807.8	35800.3	61200.0	0	347808.1
14	2	0	0	1	3	250807.8	35800.3	61200.0	0	347808.1
15	2	0	0	2	3	250807.8	35800.3	61200.0	0	347808.1
16	2	0	2	0	3	250807.8	35800.3	612000.0	0	898608.1
17	2	0	2	1	3	250807.8	35800.3	612000.0	0	898608.1
18	2	0	2	2	3	250807.8	35800.3	612000.0	0	898608.1
19	2	2	0	0	3	250807.8	35800.3	61200.0	0	347808.1
20	2	2	0	1	3	250807.8	35800.3	61200.0	0	347808.1
21	2	2	0	2	3	250807.8	35800.3	61200.0	0	347808.1
22	2	2	2	0	3	250807.8	35800.3	612000.0	0	898608.1
23	2	2	2	1	3	250807.8	35800.3	612000.0	0	898608.1
24	2	2	2	2	3	250807.8	35800.3	612000.0	0	898608.1

Table A.6: CPU time

Run	Factors				Rep.	CPU Time in sec.		
#	A	B	C	D	#	Integrated	Heuristic1	Heuristic1
1	0	0	0	0	1	1493.87	1.1	13.47
2	0	0	0	1	1	1659.94	1.24	11.57
3	0	0	0	2	1	1686.06	1.21	11.94
4	0	0	2	0	1	694.27	1.35	11.66
5	0	0	2	1	1	732.96	1.27	13.19
6	0	0	2	2	1	764.72	1.27	11.51
7	0	2	0	0	1	501.57	0.88	12.69
8	0	2	0	1	1	532.67	1.18	11.12
9	0	2	0	2	1	547.97	1.04	11.96
10	0	2	2	0	1	26.16	0.98	12.26
11	0	2	2	1	1	18.80	1.12	10.74
12	0	2	2	2	1	186.45	1.2	12.32
13	2	0	0	0	1	833.62	0.95	11.77
14	2	0	0	1	1	1166.61	1.2	11.70
15	2	0	0	2	1	852.06	1.06	11.63

Continued on next page

Table A.6 – *Continued from previous page*

Run	Factors				Rep.	CPU Time in sec.		
#	A	B	C	D	#	Integrated	Heuristic1	Heuristic1
16	2	0	2	0	1	731.29	1.23	11.94
17	2	0	2	1	1	669.54	0.96	12.80
18	2	0	2	2	1	1563.80	1.04	11.76
19	2	2	0	0	1	2216.57	1.32	10.54
20	2	2	0	1	1	580.42	1.18	13.12
21	2	2	0	2	1	759.62	1.12	12.62
22	2	2	2	0	1	1116.17	1.12	13.40
23	2	2	2	1	1	696.72	1.2	10.45
24	2	2	2	2	1	482.00	1.17	13.31
1	0	0	0	0	2	1064.44	1.77	12.17
2	0	0	0	1	2	873.79	1.17	14.18
3	0	0	0	2	2	1386.83	1.1	13.35
4	0	0	2	0	2	840.22	1.18	13.99
5	0	0	2	1	2	1102.55	1.07	12.43
6	0	0	2	2	2	660.84	1.17	12.92
7	0	2	0	0	2	516.08	1.23	11.81
8	0	2	0	1	2	561.84	1.17	14.04
9	0	2	0	2	2	465.85	1.24	14.48
10	0	2	2	0	2	144.21	1.06	12.00
11	0	2	2	1	2	545.16	1.06	13.46
12	0	2	2	2	2	85.11	0.99	12.25
13	2	0	0	0	2	1418.30	1.18	12.68
14	2	0	0	1	2	1920.82	1.13	12.65
15	2	0	0	2	2	1829.83	0.95	13.84
16	2	0	2	0	2	858.01	1.15	11.64
17	2	0	2	1	2	695.48	1.07	13.29
18	2	0	2	2	2	1145.84	1.02	14.04
19	2	2	0	0	2	861.39	1.21	12.07
20	2	2	0	1	2	735.31	1.02	12.95
21	2	2	0	2	2	1369.66	1.34	13.53
22	2	2	2	0	2	773.62	1.17	11.76
23	2	2	2	1	2	656.87	1.15	12.29
24	2	2	2	2	2	712.22	1.07	11.56
1	0	0	0	0	3	1545.49	1.17	11.08
2	0	0	0	1	3	769.71	1.24	12.67
3	0	0	0	2	3	691.68	1.41	13.90
4	0	0	2	0	3	788.44	1.1	12.01
5	0	0	2	1	3	1078.67	1.17	12.32
6	0	0	2	2	3	2616.18	0.99	11.47
7	0	2	0	0	3	1680.72	1.02	13.01
8	0	2	0	1	3	1353.96	1.17	12.04
9	0	2	0	2	3	4048.73	1.12	11.62
10	0	2	2	0	3	22.62	1.34	11.67
11	0	2	2	1	3	51.17	1.12	12.85
12	0	2	2	2	3	51.11	1.32	10.58
13	2	0	0	0	3	2022.27	0.92	12.85
14	2	0	0	1	3	1707.85	0.88	12.87

*Continued on next page*

Table A.6 – *Continued from previous page*

Run	Factors				Rep.	CPU Time in sec.		
#	A	B	C	D	#	Integrated	Heuristic1	Heuristic1
15	2	0	0	2	3	1818.29	1.07	13.62
16	2	0	2	0	3	890.17	1.02	12.84
17	2	0	2	1	3	606.03	1.09	12.01
18	2	0	2	2	3	979.73	1.01	12.57
19	2	2	0	0	3	866.15	1.17	15.32
20	2	2	0	1	3	983.65	1.31	13.68
21	2	2	0	2	3	1318.54	1.12	11.76
22	2	2	2	0	3	622.35	0.96	10.72
23	2	2	2	1	3	1125.61	1.17	11.73
24	2	2	2	2	3	375.76	1.01	11.51

## A.2 35 Flight Network

Table A.7: Cost for the schedule generated by the integrated model in 5400 sec.

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	148518.0	2452.4	86640.0	2691.6	240302.0
2	0	0	0	1	1	152188.8	2790.1	86640.0	3743.4	245362.3
3	0	0	0	2	1	147957.2	4229.7	86640.0	5137.3	243964.2
4	0	0	2	0	1	161325.4	46363.8	866400.0	7023.6	1081112.7
5	0	0	2	1	1	170436.8	2989.6	866400.0	6753.8	1046580.1
6	0	0	2	2	1	161390.6	3032.3	776400.0	1535.3	942358.2
7	0	2	0	0	1	166519.0	2397.2	86640.0	13711.8	269268.0
8	0	2	0	1	1	158405.2	45761.5	86640.0	19025.2	309831.8
9	0	2	0	2	1	165489.3	3549.1	86640.0	21644.1	277322.4
10	0	2	2	0	1	162491.0	1821.6	776400.0	3916.8	944629.5
11	0	2	2	1	1	162562.6	2007.9	776400.0	3916.8	944887.4
12	0	2	2	2	1	162488.2	3197.3	776400.0	3916.8	946002.3
13	2	0	0	0	1	302216.0	2957.1	86640.0	3025.6	394838.6
14	2	0	0	1	1	303083.8	3026.7	86640.0	3734.6	396485.0
15	2	0	0	2	1	296333.6	4973.8	86640.0	6663.5	394611.0
16	2	0	2	0	1	320871.6	2175.0	866400.0	6153.5	1195600.1
17	2	0	2	1	1	325741.8	2233.7	866400.0	8153.0	1202528.5
18	2	0	2	2	1	319384.2	3812.5	776400.0	2578.2	1102174.9
19	2	2	0	0	1	302310.2	3596.9	86640.0	12102.3	404649.4
20	2	2	0	1	1	306633.5	2168.7	86640.0	8660.6	404102.8
21	2	2	0	2	1	306616.2	2972.1	86640.0	8660.6	404888.9
22	2	2	2	0	1	326475.3	21393.5	866400.0	13147.9	1227416.6
23	2	2	2	1	1	308300.4	2446.9	866400.0	8027.7	1185174.9
24	2	2	2	2	1	322932.5	3567.8	780000.0	10312.8	1116813.1

*Continued on next page*



Table A.7 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	2	153402.4	3354.3	86640.0	1943.2	245339.9
2	0	0	0	1	2	151463.9	4059.7	86640.0	3202.5	245366.1
3	0	0	0	2	2	146706.4	6425.2	86640.0	3647.0	243418.6
4	0	0	2	0	2	153481.0	4756.9	866400.0	3637.1	1028275.0
5	0	0	2	1	2	159660.6	3749.1	776400.0	1665.8	941475.5
6	0	0	2	2	2	154129.0	5179.2	866400.0	2026.4	1027734.7
7	0	2	0	0	2	149236.2	2826.8	86640.0	7488.4	246191.4
8	0	2	0	1	2	154234.7	4007.7	86640.0	8105.7	252988.1
9	0	2	0	2	2	147057.8	22543.4	86640.0	7567.3	263808.6
10	0	2	2	0	2	162467.8	2527.9	776400.0	2589.3	943985.1
11	0	2	2	1	2	152734.6	3579.3	866400.0	10658.1	1033372.0
12	0	2	2	2	2	154478.3	34886.0	866400.0	14196.8	1069961.1
13	2	0	0	0	2	294792.6	7955.9	86640.0	2890.6	392279.0
14	2	0	0	1	2	No solution can be found in 5400 sec.				
15	2	0	0	2	2	307414.7	5021.2	86640.0	2699.7	401775.5
16	2	0	2	0	2	319369.2	3259.3	776400.0	2267.5	1101296.0
17	2	0	2	1	2	319306.3	3909.3	776400.0	2267.5	1101883.1
18	2	0	2	2	2	319264.1	5809.4	776400.0	1665.8	1103139.4
19	2	2	0	0	2	295810.7	25933.6	86640.0	11702.8	420087.1
20	2	2	0	1	2	296996.9	4243.4	86640.0	5015.0	392895.3
21	2	2	0	2	2	293342.0	6569.6	86640.0	7567.3	394118.9
22	2	2	2	0	2	322758.6	2621.5	776400.0	3777.9	1105558.1
23	2	2	2	1	2	319299.7	4019.0	776400.0	6663.1	1106381.8
24	2	2	2	2	2	322680.0	4585.4	776400.0	3777.9	1107443.4
1	0	0	0	0	3	No solution can be found in 5400 sec.				
2	0	0	0	1	3	153381.6	4454.7	86640.0	5784.0	250260.3
3	0	0	0	2	3	146678.1	6690.6	86640.0	7707.3	247716.0
4	0	0	2	0	3	162492.1	2758.7	776400.0	2413.0	944063.8
5	0	0	2	1	3	162511.7	3208.3	776400.0	2413.0	944533.0
6	0	0	2	2	3	162478.2	4557.1	776400.0	2413.0	945848.2
7	0	2	0	0	3	162500.6	2758.9	77640.0	9651.9	252551.5
8	0	2	0	1	3	149317.1	3915.4	86640.0	21302.1	261174.6
9	0	2	0	2	3	162466.6	4920.4	77640.0	9651.9	254678.9
10	0	2	2	0	3	162474.8	2759.2	776400.0	9651.9	951285.9
11	0	2	2	1	3	162485.8	3208.2	776400.0	9651.9	951746.0
12	0	2	2	2	3	162458.2	4555.4	776400.0	9651.9	953065.5
13	2	0	0	0	3	301801.5	3812.1	86640.0	9589.9	401843.4
14	2	0	0	1	3	293278.6	4612.9	86640.0	11292.8	395824.3
15	2	0	0	2	3	298899.9	15802.8	86640.0	5865.0	407207.7
16	2	0	2	0	3	320378.2	3843.8	776400.0	5569.3	1106191.3
17	2	0	2	1	3	319360.5	4585.5	776400.0	5787.1	1106133.2
18	2	0	2	2	3	319320.1	6062.8	776400.0	5409.0	1107191.8
19	2	2	0	0	3	295666.9	3682.0	86640.0	22712.7	408701.6
20	2	2	0	1	3	327317.2	3441.2	86640.0	36819.0	454217.4
21	2	2	0	2	3	298594.6	5605.9	86640.0	21302.1	412142.6
22	2	2	2	0	3	325001.2	2759.5	776400.0	9651.9	1113812.7
23	2	2	2	1	3	324951.7	3208.5	776400.0	9651.9	1114212.1

Continued on next page

Table A.7 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
24	2	2	2	2	3	324956.4	4555.5	776400.0	9651.9	1115563.8

Table A.8: Cost for the schedule generated by heuristic1

Run	Factors				Rep.	Costs					Before CSCM	
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total	Fuel	Idle Time
1	0	0	0	0	1	146714.3	2377.6	86640.0	3262.9	238994.8	146852.1	2411.7
2	0	0	0	1	1	146708.2	3028.9	86640.0	3262.9	239640.0	146825.3	3036.0
3	0	0	0	2	1	146692.2	4539.1	86640.0	3262.9	241134.2	146825.3	4539.1
4	0	0	2	0	1	159691.3	2015.1	776400.0	2578.2	940684.6	159910.8	2015.1
5	0	0	2	1	1	159684.8	2553.6	776400.0	2578.2	941216.6	159861.2	2563.1
6	0	0	2	2	1	161365.0	3032.8	776400.0	1535.3	942333.1	161597.1	3032.3
7	0	2	0	0	1	162483.4	1581.5	77640.0	3916.8	245621.7	162664.8	1581.5
8	0	2	0	1	1	162463.0	2007.9	77640.0	3916.8	246027.8	162636.4	2017.3
9	0	2	0	2	1	162453.9	2977.1	77640.0	3916.8	246987.8	162636.4	2977.1
10	0	2	2	0	1	162509.3	1581.5	776400.0	3916.8	944407.6	162716.0	1581.5
11	0	2	2	1	1	162463.0	2007.9	776400.0	3916.8	944787.8	162697.6	2017.3
12	0	2	2	2	1	162453.9	2977.1	776400.0	3916.8	945747.8	162638.8	2977.1
13	2	0	0	0	1	293427.3	2376.4	86640.0	3262.9	385706.7	293704.1	2411.7
14	2	0	0	1	1	293413.7	3026.6	86640.0	3262.9	386343.2	293650.6	3036.0
15	2	0	0	2	1	293384.5	4539.3	86640.0	3262.9	387826.8	293650.6	4539.1
16	2	0	2	0	1	319382.6	2015.1	776400.0	2578.2	1100375.9	319796.1	2050.5
17	2	0	2	1	1	319369.6	2553.7	776400.0	2578.2	1100901.5	319722.3	2563.1
18	2	0	2	2	1	319341.3	3812.5	776400.0	2578.2	1102132.0	319722.3	3812.5
19	2	2	0	0	1	295632.7	2367.5	86640.0	10827.4	395467.5	295885.3	2367.4
20	2	2	0	1	1	295620.8	3003.9	86640.0	10827.4	396092.0	295827.0	3013.3
21	2	2	0	2	1	295597.3	4483.9	86640.0	10827.4	397548.6	295831.8	4483.9
22	2	2	2	0	1	324938.3	1585.5	776400.0	3916.8	1106840.6	325346.6	1581.5
23	2	2	2	1	1	324927.4	2009.8	776400.0	3916.8	1107254.0	325277.6	2017.3
24	2	2	2	2	1	324936.7	3011.5	776400.0	3916.8	1108265.0	325280.8	3011.4
1	0	0	0	0	2	146697.0	3401.4	86640.0	1891.8	238630.2	146863.3	3401.1
2	0	0	0	1	2	146683.1	4359.8	86640.0	1891.8	239574.8	146820.6	4359.8
3	0	0	0	2	2	146649.6	6568.3	86640.0	1891.8	241749.8	146794.0	6568.2
4	0	0	2	0	2	159674.0	2901.9	776400.0	1665.8	940641.7	159889.9	2901.9
5	0	0	2	1	2	159660.7	3748.9	776400.0	1665.8	941475.4	159854.4	3748.9
6	0	0	2	2	2	161340.0	4584.6	776400.0	944.5	943269.1	161580.8	4584.6
7	0	2	0	0	2	149291.5	2943.9	86640.0	3826.4	242701.8	149413.2	2943.8
8	0	2	0	1	2	149280.1	3786.4	86640.0	3826.4	243532.9	149363.8	3786.3
9	0	2	0	2	2	149239.6	5615.9	86640.0	3826.4	245321.8	149358.3	5615.8
10	0	2	2	0	2	162464.7	2087.0	776400.0	2589.3	943541.0	162710.7	2089.7
11	0	2	2	1	2	162458.1	2638.7	776400.0	2589.3	944086.1	162630.8	2638.3
12	0	2	2	2	2	162442.4	4097.4	776400.0	2589.3	945529.0	162628.3	4097.4
13	2	0	0	0	2	293394.4	3402.2	86640.0	1891.8	385328.4	293710.2	3401.1
14	2	0	0	1	2	293366.5	4360.1	86640.0	1891.8	386258.5	293637.1	4359.8
15	2	0	0	2	2	293299.1	6568.3	86640.0	1891.8	388399.2	293593.7	6568.2
16	2	0	2	0	2	319347.8	2901.9	776400.0	1665.8	1100315.5	319779.8	2901.9
17	2	0	2	1	2	319321.3	3748.9	776400.0	1665.8	1101136.0	319695.4	3818.4
18	2	0	2	2	2	319255.5	5703.7	776400.0	1665.8	1103024.9	319665.5	5703.6
19	2	2	0	0	2	293394.4	3402.2	86640.0	7567.3	391003.9	293696.7	3401.1
20	2	2	0	1	2	293366.5	4360.1	86640.0	7567.3	391934.0	293637.1	4359.8
21	2	2	0	2	2	293299.1	6568.3	86640.0	7567.3	394074.7	293595.6	6568.2
22	2	2	2	0	2	319347.8	2901.9	776400.0	6663.1	1105312.9	319768.5	2901.9
23	2	2	2	1	2	322716.8	3093.0	776400.0	3777.9	1105987.7	323163.4	3101.0
24	2	2	2	2	2	322680.0	4584.6	776400.0	3777.9	1107442.6	323172.5	4584.6
1	0	0	0	0	3	147812.0	3680.9	86640.0	5678.2	243811.1	147938.0	3680.9
2	0	0	0	1	3	147804.2	4334.2	86640.0	5678.2	244456.6	147938.3	4334.0
3	0	0	0	2	3	147783.5	6162.3	86640.0	5678.2	246264.0	147902.8	6170.1
4	0	0	2	0	3	162474.8	2758.7	776400.0	2413.0	944046.4	162661.4	2771.4

Continued on next page

Table A.8 – Continued from previous page

Run	Factors				Rep.	Costs					Before CSCM	
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total	Fuel	Idle Time
5	0	0	2	1	3	162468.5	3208.2	776400.0	2413.0	944489.6	162735.3	3208.2
6	0	0	2	2	3	162452.3	4555.4	776400.0	2413.0	945820.7	162628.2	4563.2
7	0	2	0	0	3	162474.8	2758.7	77640.0	9651.9	252525.4	162657.0	2771.4
8	0	2	0	1	3	162468.5	3208.2	77640.0	9651.9	252968.6	162661.4	3208.2
9	0	2	0	2	3	162452.3	4555.4	77640.0	9651.9	254299.6	162624.8	4563.2
10	0	2	2	0	3	162474.8	2758.7	776400.0	9651.9	951285.4	162684.9	2758.7
11	0	2	2	1	3	162468.5	3208.2	776400.0	9651.9	951728.6	162661.4	3208.2
12	0	2	2	2	3	162452.3	4555.4	776400.0	9651.9	953059.6	162654.7	4555.3
13	2	0	0	0	3	293376.3	3978.5	86640.0	7329.2	391324.0	293696.7	3977.8
14	2	0	0	1	3	293359.6	4645.0	86640.0	7329.2	391973.8	293683.2	4644.6
15	2	0	0	2	3	293319.9	6505.5	86640.0	7329.2	393794.6	293608.1	6513.3
16	2	0	2	0	3	319332.0	3898.6	776400.0	5409.0	1105039.6	319756.9	3898.6
17	2	0	2	1	3	319316.0	4453.8	776400.0	5409.0	1105578.8	319741.4	4453.7
18	2	0	2	2	3	319277.1	6060.9	776400.0	5409.0	1107147.0	319679.9	6068.7
19	2	2	0	0	3	295624.0	3681.0	86640.0	22712.7	408657.7	295894.9	3680.9
20	2	2	0	1	3	295608.3	4334.1	86640.0	22712.7	409295.0	295865.3	4334.0
21	2	2	0	2	3	295567.0	6162.4	86640.0	22712.7	411082.1	295810.5	6170.1
22	2	2	2	0	3	324949.6	2758.7	776400.0	9651.9	1113760.2	325356.2	2758.7
23	2	2	2	1	3	324937.0	3208.2	776400.0	9651.9	1114197.1	325327.7	3208.2
24	2	2	2	2	3	324904.6	4555.4	776400.0	9651.9	1115511.9	325263.3	4563.2

Table A.9: Cost for the schedule generated by heuristic2

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	148446.3	2210.6	86640.0	2691.6	239988.6
2	0	0	0	1	1	148191.1	2759.6	86640.0	2691.6	240282.4
3	0	0	0	2	1	148178.0	4040.0	86640.0	2691.6	241549.6
4	0	0	2	0	1	161445.9	1591.4	776400.0	1535.3	940972.6
5	0	0	2	1	1	161445.2	2031.1	776400.0	1535.3	941411.7
6	0	0	2	2	1	161380.6	3067.3	776400.0	1535.3	942383.2
7	0	2	0	0	1	162486.1	1582.8	77640.0	3916.8	245625.7
8	0	2	0	1	1	162485.0	2010.3	77640.0	3916.8	246052.2
9	0	2	0	2	1	162454.4	2977.4	77640.0	3916.8	246988.6
10	0	2	2	0	1	162486.1	1582.8	776400.0	3916.8	944385.7
11	0	2	2	1	1	162467.7	2010.3	776400.0	3916.8	944794.9
12	0	2	2	2	1	162471.7	2977.4	776400.0	3916.8	945766.0
13	2	0	0	0	1	296890.8	2211.3	86640.0	2691.6	388433.7
14	2	0	0	1	1	296376.8	2759.0	86640.0	2691.6	388467.4
15	2	0	0	2	1	296350.8	4039.4	86640.0	2691.6	389721.8
16	2	0	2	0	1	322888.8	1592.2	776400.0	1535.3	1102416.3
17	2	0	2	1	1	322889.4	2033.3	776400.0	1535.3	1102858.1
18	2	0	2	2	1	322759.4	3068.7	776400.0	1535.3	1103763.4
19	2	2	0	0	1	298593.3	2202.9	86640.0	8542.2	395978.5
20	2	2	0	1	1	299084.7	2736.5	86640.0	8542.2	397003.4
21	2	2	0	2	1	299064.6	3984.1	86640.0	8542.2	398230.9
22	2	2	2	0	1	324966.8	1581.9	776400.0	3916.8	1106865.5
23	2	2	2	1	1	324926.6	2009.1	776400.0	3916.8	1107252.5
24	2	2	2	2	1	324943.6	2979.4	776400.0	3916.8	1108239.8
1	0	0	0	0	2	148178.2	3386.8	86640.0	1253.8	239458.8

Continued on next page

Table A.9 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
2	0	0	0	1	2	148165.9	4242.9	86640.0	1253.8	240302.5
3	0	0	0	2	2	149246.5	5616.4	86640.0	956.6	242459.6
4	0	0	2	0	2	161367.0	2528.3	776400.0	944.5	941239.7
5	0	0	2	1	2	161427.1	3093.5	776400.0	944.5	941865.1
6	0	0	2	2	2	161428.7	4586.1	776400.0	944.5	943359.2
7	0	2	0	0	2	149293.3	2945.3	86640.0	3826.4	242705.0
8	0	2	0	1	2	149531.5	3787.0	86640.0	3826.4	243784.9
9	0	2	0	2	2	149246.5	5616.4	86640.0	3826.4	245329.4
10	0	2	2	0	2	162486.5	2089.0	776400.0	2589.3	943564.8
11	0	2	2	1	2	162458.8	2638.7	776400.0	2589.3	944086.8
12	0	2	2	2	2	162464.2	4099.8	776400.0	2589.3	945553.3
13	2	0	0	0	2	296350.9	3386.0	86640.0	1253.8	387630.6
14	2	0	0	1	2	296326.2	4242.0	86640.0	1253.8	388462.0
15	2	0	0	2	2	296970.1	6218.2	86640.0	1253.8	391082.0
16	2	0	2	0	2	322732.8	2528.6	776400.0	944.5	1102605.8
17	2	0	2	1	2	322852.4	3095.8	776400.0	944.5	1103292.7
18	2	0	2	2	2	322851.9	4587.2	776400.0	944.5	1104783.5
19	2	2	0	0	2	296350.9	3386.0	86640.0	5015.0	391391.9
20	2	2	0	1	2	296326.2	4242.0	86640.0	5015.0	392223.3
21	2	2	0	2	2	296263.6	6218.5	86640.0	5015.0	394137.1
22	2	2	2	0	2	322856.1	2531.0	776400.0	3777.9	1105565.1
23	2	2	2	1	2	322717.2	3093.8	776400.0	3777.9	1105988.9
24	2	2	2	2	2	322851.9	4587.2	776400.0	3777.9	1107616.9
1	0	0	0	0	3	149295.0	3365.1	86640.0	5325.5	244625.6
2	0	0	0	1	3	149288.0	3915.8	86640.0	5325.5	245169.3
3	0	0	0	2	3	149268.3	5604.5	86640.0	5325.5	246838.3
4	0	0	2	0	3	162475.3	2758.9	776400.0	2413.0	944047.2
5	0	0	2	1	3	162469.0	3208.4	776400.0	2413.0	944490.4
6	0	0	2	2	3	162453.1	4555.7	776400.0	2413.0	945821.8
7	0	2	0	0	3	162494.5	2759.6	77640.0	9651.9	252546.0
8	0	2	0	1	3	162487.4	3208.9	77640.0	9651.9	252988.2
9	0	2	0	2	3	162453.1	4555.7	77640.0	9651.9	254300.7
10	0	2	2	0	3	162494.5	2759.6	776400.0	9651.9	951306.0
11	0	2	2	1	3	162469.0	3208.4	776400.0	9651.9	951729.4
12	0	2	2	2	3	162453.1	4555.7	776400.0	9651.9	953060.7
13	2	0	0	0	3	296339.8	3662.6	86640.0	6976.5	393618.9
14	2	0	0	1	3	296324.8	4226.9	86640.0	6976.5	394168.2
15	2	0	0	2	3	296287.0	5948.1	86640.0	6976.5	395851.6
16	2	0	2	0	3	322762.4	3058.4	776400.0	4064.0	1106284.8
17	2	0	2	1	3	322749.1	3521.6	776400.0	4064.0	1106734.7
18	2	0	2	2	3	322729.5	4899.4	776400.0	4064.0	1108092.8
19	2	2	0	0	3	298587.7	3366.1	86640.0	21302.1	409895.9
20	2	2	0	1	3	298573.6	3916.7	86640.0	21302.1	410432.4
21	2	2	0	2	3	298533.9	5605.2	86640.0	21302.1	412081.3
22	2	2	2	0	3	324985.4	2761.8	776400.0	9651.9	1113799.1
23	2	2	2	1	3	324937.5	3209.4	776400.0	9651.9	1114198.8
24	2	2	2	2	3	324905.0	4556.5	776400.0	9651.9	1115513.4

Table A.10: Cost for the schedule generated by heuristic2 at the root node

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	162472.7	1583.8	77640.0	979.2	242675.8
2	0	0	0	1	1	162494.7	2009.0	77640.0	979.2	243122.9
3	0	0	0	2	1	162454.4	2977.4	77640.0	979.2	244051.0
4	0	0	2	0	1	162511.5	1582.5	776400.0	979.2	941473.2
5	0	0	2	1	1	162499.4	2009.6	776400.0	979.2	941888.2
6	0	0	2	2	1	162471.7	2977.4	776400.0	979.2	942828.3
7	0	2	0	0	1	162486.1	1582.8	77640.0	3916.8	245625.7
8	0	2	0	1	1	162485.0	2010.3	77640.0	3916.8	246052.2
9	0	2	0	2	1	162454.4	2977.4	77640.0	3916.8	246988.6
10	0	2	2	0	1	162486.1	1582.8	776400.0	3916.8	944385.7
11	0	2	2	1	1	162467.7	2010.3	776400.0	3916.8	944794.9
12	0	2	2	2	1	162471.7	2977.4	776400.0	3916.8	945766.0
13	2	0	0	0	1	324936.1	1582.3	77640.0	979.2	405137.7
14	2	0	0	1	1	324985.0	2008.3	77640.0	979.2	405612.5
15	2	0	0	2	1	324908.3	2978.4	77640.0	979.2	406505.9
16	2	0	2	0	1	325040.6	1583.0	776400.0	979.2	1104002.8
17	2	0	2	1	1	324992.2	2009.2	776400.0	979.2	1104380.6
18	2	0	2	2	1	324943.6	2979.4	776400.0	979.2	1105302.2
19	2	2	0	0	1	324966.8	1581.9	77640.0	3916.8	408105.5
20	2	2	0	1	1	324962.1	2011.4	77640.0	3916.8	408530.3
21	2	2	0	2	1	324908.3	2978.4	77640.0	3916.8	409443.5
22	2	2	2	0	1	324966.8	1581.9	776400.0	3916.8	1106865.5
23	2	2	2	1	1	324926.6	2009.1	776400.0	3916.8	1107252.5
24	2	2	2	2	1	324943.6	2979.4	776400.0	3916.8	1108239.8
1	0	0	0	0	2	162467.8	2088.5	77640.0	647.3	242843.6
2	0	0	0	1	2	162458.8	2638.7	77640.0	647.3	243384.9
3	0	0	0	2	2	162446.5	4099.5	77640.0	647.3	244833.3
4	0	0	2	0	2	162467.8	2088.5	776400.0	647.3	941603.6
5	0	0	2	1	2	162476.3	2638.8	776400.0	647.3	942162.4
6	0	0	2	2	2	162446.5	4099.5	776400.0	647.3	943593.3
7	0	2	0	0	2	162467.8	2088.5	77640.0	2589.3	244785.6
8	0	2	0	1	2	162458.8	2638.7	77640.0	2589.3	245326.8
9	0	2	0	2	2	162464.2	4099.8	77640.0	2589.3	246793.3
10	0	2	2	0	2	162486.5	2089.0	776400.0	2589.3	943564.8
11	0	2	2	1	2	162458.8	2638.7	776400.0	2589.3	944086.8
12	0	2	2	2	2	162464.2	4099.8	776400.0	2589.3	945553.3
13	2	0	0	0	2	324929.9	2087.8	77640.0	647.3	405305.0
14	2	0	0	1	2	324916.0	2638.9	77640.0	647.3	405842.2
15	2	0	0	2	2	324886.3	4100.9	77640.0	647.3	407274.5
16	2	0	2	0	2	324929.9	2087.8	776400.0	647.3	1104065.0
17	2	0	2	1	2	324951.6	2640.7	776400.0	647.3	1104639.6
18	2	0	2	2	2	324886.3	4100.9	776400.0	647.3	1106034.5
19	2	2	0	0	2	324929.9	2087.8	77640.0	2589.3	407247.0

*Continued on next page*

Table A.10 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
20	2	2	0	1	2	324916.0	2638.9	77640.0	2589.3	407784.1
21	2	2	0	2	2	324920.3	4100.0	77640.0	2589.3	409249.6
22	2	2	2	0	2	324965.2	2088.9	776400.0	2589.3	1106043.3
23	2	2	2	1	2	324916.0	2638.9	776400.0	2589.3	1106544.1
24	2	2	2	2	2	324920.3	4100.0	776400.0	2589.3	1108009.6
1	0	0	0	0	3	162475.3	2758.9	77640.0	2413.0	245287.2
2	0	0	0	1	3	162469.0	3208.4	77640.0	2413.0	245730.4
3	0	0	0	2	3	162453.1	4555.7	77640.0	2413.0	247061.8
4	0	0	2	0	3	162475.3	2758.9	776400.0	2413.0	944047.2
5	0	0	2	1	3	162469.0	3208.4	776400.0	2413.0	944490.4
6	0	0	2	2	3	162453.1	4555.7	776400.0	2413.0	945821.8
7	0	2	0	0	3	168920.6	3076.9	78240.0	8290.7	258528.2
8	0	2	0	1	3	168930.6	3628.8	78240.0	8290.7	259090.1
9	0	2	0	2	3	168913.4	5115.6	78240.0	8290.7	260559.7
10	0	2	2	0	3	162494.5	2759.6	776400.0	9651.9	951306.0
11	0	2	2	1	3	162469.0	3208.4	776400.0	9651.9	951729.4
12	0	2	2	2	3	162453.1	4555.7	776400.0	9651.9	953060.7
13	2	0	0	0	3	324971.8	2760.2	77640.0	2413.0	407785.0
14	2	0	0	1	3	324937.5	3209.4	77640.0	2413.0	408199.9
15	2	0	0	2	3	324905.0	4556.5	77640.0	2413.0	409514.5
16	2	0	2	0	3	324971.8	2760.2	776400.0	2413.0	1106545.0
17	2	0	2	1	3	324937.5	3209.4	776400.0	2413.0	1106959.9
18	2	0	2	2	3	324905.0	4556.5	776400.0	2413.0	1108274.5
19	2	2	0	0	3	337835.8	3075.5	78240.0	8290.7	427442.0
20	2	2	0	1	3	337855.5	3630.6	78240.0	8290.7	428016.8
21	2	2	0	2	3	337821.9	5115.0	78240.0	8290.7	429467.6
22	2	2	2	0	3	324985.4	2761.8	776400.0	9651.9	1113799.1
23	2	2	2	1	3	324937.5	3209.4	776400.0	9651.9	1114198.8
24	2	2	2	2	3	324905.0	4556.5	776400.0	9651.9	1115513.4

Table A.11: CPU time

Run	Factors				Rep.	CPU Time in sec.		
#	A	B	C	D	#	Integrated	Heuristic1	Heuristic1
1	0	0	0	0	1	5400	7.64	40.79
2	0	0	0	1	1	5400	6.07	46.30
3	0	0	0	2	1	5400	7.64	42.93
4	0	0	2	0	1	5400	7.52	37.96
5	0	0	2	1	1	5400	3.24	41.86
6	0	0	2	2	1	5400	4.07	40.67
7	0	2	0	0	1	5400	6.60	42.17
8	0	2	0	1	1	5400	7.00	36.13
9	0	2	0	2	1	5400	6.27	49.11

Continued on next page

Table A.11 – *Continued from previous page*

Run	Factors				Rep.	CPU Time in sec.		
#	A	B	C	D	#	Integrated	Heuristic1	Heuristic1
10	0	2	2	0	1	5400	5.71	32.82
11	0	2	2	1	1	5400	6.01	33.82
12	0	2	2	2	1	5400	4.12	37.86
13	2	0	0	0	1	5400	6.47	41.90
14	2	0	0	1	1	5400	4.66	43.81
15	2	0	0	2	1	5400	5.26	48.86
16	2	0	2	0	1	5400	4.24	39.66
17	2	0	2	1	1	5400	4.63	37.69
18	2	0	2	2	1	5400	4.15	39.34
19	2	2	0	0	1	5400	7.80	36.47
20	2	2	0	1	1	5400	6.79	40.72
21	2	2	0	2	1	5400	5.32	40.67
22	2	2	2	0	1	5400	7.24	33.15
23	2	2	2	1	1	5400	5.18	31.64
24	2	2	2	2	1	5400	5.26	39.61
1	0	0	0	0	2	5400	5.63	42.43
2	0	0	0	1	2	5400	6.05	54.71
3	0	0	0	2	2	5400	5.49	41.22
4	0	0	2	0	2	5400	5.82	44.51
5	0	0	2	1	2	5400	4.95	41.54
6	0	0	2	2	2	5400	6.21	39.89
7	0	2	0	0	2	5400	6.44	38.95
8	0	2	0	1	2	5400	4.82	46.96
9	0	2	0	2	2	5400	4.73	44.09
10	0	2	2	0	2	5400	4.95	44.62
11	0	2	2	1	2	5400	6.52	38.20
12	0	2	2	2	2	5400	4.01	42.82
13	2	0	0	0	2	5400	6.13	41.90
14	2	0	0	1	2	No solution	6.29	47.24
15	2	0	0	2	2	5400	6.01	42.49
16	2	0	2	0	2	5400	5.65	38.39
17	2	0	2	1	2	5400	4.63	42.70
18	2	0	2	2	2	5400	4.38	41.23
19	2	2	0	0	2	5400	6.90	40.05
20	2	2	0	1	2	5400	5.85	41.17
21	2	2	0	2	2	5400	5.55	45.60
22	2	2	2	0	2	5400	5.93	36.94
23	2	2	2	1	2	5400	5.96	39.86
24	2	2	2	2	2	5400	6.41	40.79
1	0	0	0	0	3	No solution	5.41	46.85
2	0	0	0	1	3	5400	4.88	40.79
3	0	0	0	2	3	5400	7.18	40.40
4	0	0	2	0	3	5400	5.85	40.65
5	0	0	2	1	3	5400	6.88	41.84
6	0	0	2	2	3	5400	5.57	36.13
7	0	2	0	0	3	5400	4.37	38.19
8	0	2	0	1	3	5400	5.74	36.58

*Continued on next page*

Table A.11 – Continued from previous page

Run	Factors				Rep.	CPU Time in sec.		
#	A	B	C	D	#	Integrated	Heuristic1	Heuristic1
9	0	2	0	2	3	5400	7.22	34.29
10	0	2	2	0	3	5235.64	4.23	43.74
11	0	2	2	1	3	5400	5.10	34.98
12	0	2	2	2	3	5400	5.24	36.08
13	2	0	0	0	3	5400	5.35	42.46
14	2	0	0	1	3	5400	6.10	36.15
15	2	0	0	2	3	5400	6.16	36.44
16	2	0	2	0	3	5400	5.46	39.64
17	2	0	2	1	3	5400	6.22	42.18
18	2	0	2	2	3	5400	5.69	38.27
19	2	2	0	0	3	5400	5.91	38.10
20	2	2	0	1	3	5400	6.08	34.32
21	2	2	0	2	3	5400	6.33	37.71
22	2	2	2	0	3	5400	4.49	33.85
23	2	2	2	1	3	4243.37	4.17	35.62
24	2	2	2	2	3	5400	6.69	35.10

## A.3 114 Flight Network

Table A.12: Cost for the schedule generated by heuristic1

Run	Factors				Rep.	Costs					Before CSCM	
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total	Fuel	Idle Time
1	0	0	0	0	1	495008.2	7806.5	248460.0	10636.1	761910.8	495679.4	7813.3
2	0	0	0	1	1	494734.1	9300.1	248460.0	10636.1	763130.3	495433.4	9341.9
3	0	0	0	2	1	495980.8	13478.1	248460.0	11158.4	769077.3	496689.3	13604.4
4	0	0	2	0	1	495510.1	8785.8	2484600.0	9472.3	2998368.2	496218.7	8785.4
5	0	0	2	1	1	494771.3	9300.2	2484600.0	10636.1	2999307.6	495457.4	9341.9
6	0	0	2	2	1	494790.1	13462.7	2484600.0	10797.2	3003649.9	495544.6	13459.6
7	0	2	0	0	1	506198.1	7761.2	248460.0	18121.8	780541.1	506935.7	8086.0
8	0	2	0	1	1	506695.1	9656.4	248460.0	17710.8	782522.3	507284.5	10023.7
9	0	2	0	2	1	506592.0	13662.7	248460.0	17710.8	786425.6	507119.9	13939.3
10	0	2	2	0	1	506189.3	7761.3	2484600.0	18121.8	3016672.4	506904.7	8084.0
11	0	2	2	1	1	506695.1	9656.4	2484600.0	17710.8	3018662.3	507284.5	10023.7
12	0	2	2	2	1	506592.0	13662.7	2484600.0	17710.8	3022565.6	507119.9	13939.3
13	2	0	0	0	1	985949.0	8931.6	248460.0	12965.8	1256306.4	987254.3	8954.4
14	2	0	0	1	1	986054.7	9968.2	248460.0	11582.6	1256065.5	987407.1	10522.6
15	2	0	0	2	1	985692.6	14785.7	248460.0	11582.6	1260520.9	987195.0	15178.0
16	2	0	2	0	1	985483.4	8851.8	2484600.0	12138.7	3491073.9	986856.9	8926.4
17	2	0	2	1	1	985929.6	9968.2	2484600.0	11582.6	3492080.4	987270.4	10522.6
18	2	0	2	2	1	985779.6	14417.2	2484600.0	11799.6	3496596.4	987293.4	14857.7
19	2	2	0	0	1	980783.2	8666.7	257280.0	44169.3	1290899.2	982340.9	8807.6
20	2	2	0	1	1	991149.4	9868.7	257280.0	30893.2	1289191.3	992404.3	9867.4
21	2	2	0	2	1	988098.1	13937.1	257280.0	33457.9	1292773.0	989488.6	13952.2
22	2	2	2	0	1	992834.6	10189.5	2484600.0	32920.3	3520544.4	993978.4	10919.7
23	2	2	2	1	1	995611.5	12216.3	2484600.0	30355.6	3522783.4	996833.3	12802.8
24	2	2	2	2	1	1000892.4	14161.8	2484600.0	27985.8	3527640.1	1002181.6	14225.9
1	0	0	0	0	2	497218.0	8310.7	248460.0	8749.2	762737.9	497900.7	8310.5

Continued on next page



Table A.12 – Continued from previous page

Run	Factors				Rep.	Costs					Before CSCM	
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total	Fuel	Idle Time
2	0	0	0	1	2	493255.2	10373.4	248460.0	9286.8	761375.3	494036.8	10373.0
3	0	0	0	2	2	496420.0	13197.3	248460.0	9643.1	767720.4	497236.7	13197.0
4	0	0	2	0	2	496665.1	8625.4	2484600.0	10235.3	3000125.8	497511.0	8625.0
5	0	0	2	1	2	493301.5	10321.1	2484600.0	9286.8	2997509.3	494071.6	10320.6
6	0	0	2	2	2	496737.4	14865.3	2484600.0	10151.5	3006354.2	497593.1	14926.3
7	0	2	0	0	2	499753.1	12438.1	248460.0	15222.6	775873.8	500759.1	12577.6
8	0	2	0	1	2	501618.0	12708.4	248460.0	14752.4	777538.8	502493.1	12708.1
9	0	2	0	2	2	500600.2	17495.9	248460.0	15222.6	781778.8	501438.3	18151.7
10	0	2	2	0	2	498735.5	9364.7	2484600.0	18499.1	3011199.4	499725.4	9364.4
11	0	2	2	1	2	502023.0	13342.6	2484600.0	12908.7	3012874.2	502826.3	13644.4
12	0	2	2	2	2	501800.2	16537.7	2484600.0	14752.4	3017690.3	502676.5	16537.3
13	2	0	0	0	2	983727.0	12378.0	248460.0	10182.3	1254747.3	985011.5	12543.0
14	2	0	0	1	2	973424.3	13452.5	257280.0	10273.2	1254429.9	974745.5	13451.3
15	2	0	0	2	2	985916.9	15881.0	248460.0	8217.7	1258475.6	987671.8	16048.4
16	2	0	2	0	2	983709.3	10906.9	2484600.0	9983.4	3489199.6	985378.0	10952.0
17	2	0	2	1	2	985692.3	11718.4	2484600.0	7918.9	3489929.7	987415.1	11717.9
18	2	0	2	2	2	985773.3	19240.9	2484600.0	11206.5	3500820.7	987756.8	20566.4
19	2	2	0	0	2	986929.3	12714.4	257280.0	19105.7	1276029.5	988350.9	12802.1
20	2	2	0	1	2	982762.4	14438.8	257280.0	24956.7	1279438.0	984064.7	14505.5
21	2	2	0	2	2	983982.7	20020.9	257280.0	22227.7	1283511.3	985210.0	20191.8
22	2	2	2	0	2	987540.3	14080.1	2484600.0	27692.4	3513912.7	988722.3	14244.1
23	2	2	2	1	2	998061.6	10657.3	2484600.0	18499.1	3511818.1	999832.0	10701.9
24	2	2	2	2	2	987499.1	20539.8	2484600.0	27605.2	3520244.1	988980.0	20613.5
1	0	0	0	0	3	493950.1	10143.3	248460.0	24312.3	776865.7	494760.6	10207.3
2	0	0	0	1	3	493262.7	11164.2	248460.0	26124.2	779011.1	494000.2	11213.6
3	0	0	0	2	3	499132.6	14171.9	248460.0	24150.8	785915.2	499716.4	14170.9
4	0	0	2	0	3	493436.9	10792.0	2484600.0	23797.7	3012626.5	494033.1	10992.9
5	0	0	2	1	3	495868.1	11437.9	2484600.0	24139.3	3016045.2	496594.7	11453.1
6	0	0	2	2	3	495912.4	15547.7	2484600.0	25578.4	3021638.4	496569.8	15563.2
7	0	2	0	0	3	522714.6	12525.5	248280.0	34376.0	817896.1	523065.8	13379.5
8	0	2	0	1	3	522689.7	14398.3	248280.0	34376.0	819744.0	523076.2	15024.4
9	0	2	0	2	3	521574.7	20204.7	248280.0	34917.7	824977.1	522143.7	20204.3
10	0	2	2	0	3	522714.5	12525.3	2482800.0	34376.0	3052415.8	523065.8	13379.5
11	0	2	2	1	3	522689.6	14398.1	2482800.0	34376.0	3054263.8	523068.2	15024.4
12	0	2	2	2	3	521573.1	20204.6	2482800.0	34917.7	3059495.4	522143.7	20204.3
13	2	0	0	0	3	973543.2	10674.0	257280.0	28962.1	1270459.4	974876.2	10692.6
14	2	0	0	1	3	986934.5	12699.3	248460.0	23725.4	1271819.2	988303.6	12804.2
15	2	0	0	2	3	974976.1	17478.4	257280.0	27453.8	1277188.3	976005.6	17610.0
16	2	0	2	0	3	986875.8	10792.0	2484600.0	23853.6	3506121.4	988189.0	10985.1
17	2	0	2	1	3	987094.4	12734.1	2484600.0	23912.4	3508340.9	988450.1	12886.2
18	2	0	2	2	3	986653.9	17710.6	2484600.0	23939.1	3512903.6	987969.6	17782.7
19	2	2	0	0	3	999687.7	15198.8	257280.0	60596.2	1332762.7	1000814.6	15471.1
20	2	2	0	1	3	1007135.7	17576.9	248460.0	57940.3	1331112.9	1008110.8	17793.9
21	2	2	0	2	3	1004909.4	22277.7	248460.0	60156.4	1335803.5	1006256.2	22293.3
22	2	2	2	0	3	1011701.3	13650.6	2484600.0	57992.1	3567944.0	1012982.5	13922.0
23	2	2	2	1	3	1012129.5	15934.3	2484600.0	57295.9	3569959.7	1013377.6	16108.2
24	2	2	2	2	3	1013251.2	17680.7	2484600.0	55255.7	3570787.5	1014296.0	17696.0

Table A.13: Cost for the schedule generated by heuristic2

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	534860.0	5332.2	248580.0	2527.2	791299.4
2	0	0	0	1	1	531543.9	6384.8	248100.0	2846.4	788875.1
3	0	0	0	2	1	535839.1	8704.8	248100.0	3690.6	796334.5
4	0	0	2	0	1	532692.3	5067.8	2481000.0	2980.4	3021740.4
5	0	0	2	1	1	532152.8	6390.1	2481000.0	2846.4	3022389.2

Continued on next page

Table A.13 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
6	0	0	2	2	1	535297.6	9262.3	2481000.0	4373.9	3029933.8
7	0	2	0	0	1	535443.6	8079.2	248100.0	5465.5	797088.2
8	0	2	0	1	1	535375.6	9609.3	248100.0	4740.5	797825.4
9	0	2	0	2	1	535672.4	13077.0	248100.0	5717.0	802566.4
10	0	2	2	0	1	536532.7	7661.7	2481000.0	4740.5	3029934.9
11	0	2	2	1	1	535480.5	9891.9	2481000.0	5465.5	3031837.9
12	0	2	2	2	1	536139.8	13081.6	2481000.0	5717.0	3035938.4
13	2	0	0	0	1	1070297.8	5129.0	248580.0	2869.6	1326876.5
14	2	0	0	1	1	1062631.1	6389.7	248100.0	2846.4	1319967.2
15	2	0	0	2	1	1071673.7	8709.9	248100.0	3690.6	1332174.2
16	2	0	2	0	1	1065362.7	5068.0	2481000.0	2980.4	3554411.0
17	2	0	2	1	1	1064283.8	6386.7	2481000.0	2846.4	3554516.8
18	2	0	2	2	1	1070583.4	9263.2	2481000.0	4373.9	3565220.5
19	2	2	0	0	1	1070875.3	8084.9	248100.0	5465.5	1332525.6
20	2	2	0	1	1	1070749.2	9619.5	248100.0	4740.5	1333209.2
21	2	2	0	2	1	1071341.1	13082.2	248100.0	5717.0	1338240.3
22	2	2	2	0	1	1071498.5	8080.6	2481000.0	5465.5	3566044.5
23	2	2	2	1	1	1070874.3	9555.0	2481000.0	5465.5	3566894.8
24	2	2	2	2	1	1072255.8	13077.3	2481000.0	5717.0	3572050.1
1	0	0	0	0	2	528271.3	8241.5	248100.0	1758.5	786371.2
2	0	0	0	1	2	528899.1	8490.6	248100.0	2079.0	787568.7
3	0	0	0	2	2	534095.8	11600.2	248100.0	2243.9	796039.8
4	0	0	2	0	2	525076.1	8470.9	2481000.0	1916.3	3016463.3
5	0	0	2	1	2	528542.1	8747.9	2481000.0	2352.4	3020642.3
6	0	0	2	2	2	534127.1	11601.2	2481000.0	2243.9	3028972.1
7	0	2	0	0	2	532494.6	9647.6	248580.0	3850.2	794572.4
8	0	2	0	1	2	533122.6	11131.8	248580.0	3850.2	796684.7
9	0	2	0	2	2	533142.6	14689.6	248100.0	4039.3	799971.5
10	0	2	2	0	2	529433.6	9492.4	2481000.0	4943.5	3024869.5
11	0	2	2	1	2	530037.1	10741.4	2481000.0	4943.5	3026722.0
12	0	2	2	2	2	533152.5	14689.7	2481000.0	4039.3	3032881.4
13	2	0	0	0	2	1056536.3	8251.8	248100.0	1758.5	1314646.6
14	2	0	0	1	2	1057793.7	8492.7	248100.0	2079.0	1316465.4
15	2	0	0	2	2	1068191.4	11609.3	248100.0	2243.9	1330144.5
16	2	0	2	0	2	1050143.5	8470.2	2481000.0	1916.3	3541530.0
17	2	0	2	1	2	1057065.1	8745.5	2481000.0	2352.4	3549162.9
18	2	0	2	2	2	1068250.0	11609.4	2481000.0	2243.9	3563103.3
19	2	2	0	0	2	1064977.6	9647.7	248580.0	3850.2	1327055.6
20	2	2	0	1	2	1066218.8	11135.3	248580.0	3850.2	1329784.3
21	2	2	0	2	2	1066281.4	14690.5	248100.0	4039.3	1333111.2
22	2	2	2	0	2	1058863.8	9497.4	2481000.0	4943.5	3554304.7
23	2	2	2	1	2	1060076.3	10751.0	2481000.0	4943.5	3556770.8
24	2	2	2	2	2	1066300.5	14689.8	2481000.0	4039.3	3566029.6
1	0	0	0	0	3	532968.5	7779.8	248100.0	9222.8	798071.1
2	0	0	0	1	3	532732.1	9082.7	248100.0	9440.7	799355.4
3	0	0	0	2	3	533979.4	12024.2	248100.0	9603.4	803707.0
4	0	0	2	0	3	533034.6	8003.4	2481000.0	8994.3	3031032.3

Continued on next page

Table A.13 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
5	0	0	2	1	3	532900.3	9082.3	2481000.0	9440.7	3032423.3
6	0	0	2	2	3	533980.1	12024.5	2481000.0	9603.4	3036608.1
7	0	2	0	0	3	542723.0	12296.9	248700.0	24684.7	828404.5
8	0	2	0	1	3	540277.1	15291.7	248700.0	25147.4	829416.2
9	0	2	0	2	3	539493.6	19931.7	248700.0	26058.2	834183.6
10	0	2	2	0	3	536267.4	11723.0	2481000.0	26197.1	3055187.4
11	0	2	2	1	3	533776.6	14757.4	2481000.0	26659.9	3056193.9
12	0	2	2	2	3	533024.7	19330.7	2481000.0	27570.7	3060926.1
13	2	0	0	0	3	1065905.2	7774.1	248100.0	9222.8	1331002.2
14	2	0	0	1	3	1065460.5	9084.7	248100.0	9440.7	1332085.9
15	2	0	0	2	3	1066623.1	13164.3	248100.0	9440.7	1337328.1
16	2	0	2	0	3	1065580.0	7773.4	2481000.0	9222.8	3563576.3
17	2	0	2	1	3	1065798.1	9083.9	2481000.0	9440.7	3565322.7
18	2	0	2	2	3	1066741.7	13165.0	2481000.0	9440.7	3570347.3
19	2	2	0	0	3	1079892.6	13521.4	248700.0	25147.4	1367261.4
20	2	2	0	1	3	1080539.8	15290.4	248700.0	25147.4	1369677.6
21	2	2	0	2	3	1078957.3	19933.4	248700.0	26058.2	1373648.9
22	2	2	2	0	3	1067766.6	12936.8	2481000.0	26659.9	3588363.4
23	2	2	2	1	3	1067530.3	14762.0	2481000.0	26659.9	3589952.2
24	2	2	2	2	3	1066045.7	19331.7	2481000.0	27570.7	3593948.1

Table A.14: Cost for the schedule generated by heuristic2 at the root node

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	540184.0	4423.2	248580.0	2554.4	795741.5
2	0	0	0	1	1	539161.3	6244.2	248100.0	2587.0	796092.5
3	0	0	0	2	1	538028.5	8182.5	248100.0	3690.6	798001.6
4	0	0	2	0	1	538415.3	5008.6	2481000.0	2665.1	3027089.0
5	0	0	2	1	1	539059.8	6237.9	2481000.0	2587.0	3028884.7
6	0	0	2	2	1	537371.1	8179.0	2481000.0	3690.6	3030240.7
7	0	2	0	0	1	538250.9	6916.9	248100.0	4740.5	798008.3
8	0	2	0	1	1	537432.4	8598.6	248100.0	4740.5	798871.4
9	0	2	0	2	1	538931.5	11733.5	248100.0	5717.0	804482.0
10	0	2	2	0	1	539222.3	6914.3	2481000.0	4740.5	3031877.1
11	0	2	2	1	1	540258.0	7882.7	2481000.0	5608.5	3034749.2
12	0	2	2	2	1	538283.1	11737.2	2481000.0	5717.0	3036737.4
13	2	0	0	0	1	1080359.8	4430.4	248580.0	2554.4	1335924.6
14	2	0	0	1	1	1078289.1	6238.3	248100.0	2587.0	1335214.4
15	2	0	0	2	1	1076036.5	8182.8	248100.0	3690.6	1336009.9
16	2	0	2	0	1	1076806.3	5006.1	2481000.0	2665.1	3565477.6
17	2	0	2	1	1	1078114.0	6239.1	2481000.0	2587.0	3567940.2
18	2	0	2	2	1	1074737.1	8184.1	2481000.0	3690.6	3567611.8

Continued on next page

Table A.14 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
19	2	2	0	0	1	1076481.4	6920.5	248100.0	4740.5	1336242.4
20	2	2	0	1	1	1074856.2	8600.2	248100.0	4740.5	1336296.8
21	2	2	0	2	1	1077858.1	11743.3	248100.0	5717.0	1343418.4
22	2	2	2	0	1	1078432.1	6914.1	2481000.0	4740.5	3571086.7
23	2	2	2	1	1	1080505.8	7884.8	2481000.0	5608.5	3574999.1
24	2	2	2	2	1	1076564.5	11737.5	2481000.0	5717.0	3575019.0
1	0	0	0	0	2	535007.8	7124.7	248100.0	1758.5	791991.0
2	0	0	0	1	2	535620.5	7827.0	248100.0	2133.3	793680.7
3	0	0	0	2	2	536679.2	10745.7	248100.0	2243.9	797768.8
4	0	0	2	0	2	535310.9	7127.1	2481000.0	1758.5	3025196.5
5	0	0	2	1	2	535174.4	7825.9	2481000.0	2133.3	3026133.6
6	0	0	2	2	2	536696.8	10747.3	2481000.0	2243.9	3030687.9
7	0	2	0	0	2	535956.1	9531.0	248580.0	2946.0	797013.2
8	0	2	0	1	2	541273.3	9935.5	248580.0	3850.2	803639.1
9	0	2	0	2	2	537207.1	12830.9	248100.0	4943.5	803081.5
10	0	2	2	0	2	534444.9	10455.8	2481000.0	2921.4	3028822.1
11	0	2	2	1	2	537862.3	9551.6	2481000.0	4943.5	3033357.4
12	0	2	2	2	2	537318.3	12828.2	2481000.0	4943.5	3036090.0
13	2	0	0	0	2	1070008.0	7126.8	248100.0	1758.5	1326993.3
14	2	0	0	1	2	1071224.7	7830.6	248100.0	2133.3	1329288.6
15	2	0	0	2	2	1073353.8	10746.2	248100.0	2243.9	1334443.8
16	2	0	2	0	2	1070607.1	7126.0	2481000.0	1758.5	3560491.5
17	2	0	2	1	2	1070335.8	7827.8	2481000.0	2133.3	3561296.9
18	2	0	2	2	2	1073383.2	10749.8	2481000.0	2243.9	3567376.9
19	2	2	0	0	2	1071899.8	9529.9	248580.0	2946.0	1332955.7
20	2	2	0	1	2	1082541.5	9944.1	248580.0	3850.2	1344915.8
21	2	2	0	2	2	1074401.0	12828.8	248100.0	4943.5	1340273.3
22	2	2	2	0	2	1068875.7	10454.0	2481000.0	2921.4	3563251.1
23	2	2	2	1	2	1075719.4	9555.2	2481000.0	4943.5	3571218.1
24	2	2	2	2	2	1074633.6	12831.9	2481000.0	4943.5	3573409.0
1	0	0	0	0	3	535791.6	7526.3	248100.0	8700.5	800118.4
2	0	0	0	1	3	536008.0	8609.8	248100.0	9149.7	801867.4
3	0	0	0	2	3	534068.9	12029.1	248100.0	9603.4	803801.4
4	0	0	2	0	3	536062.1	7521.3	2481000.0	8703.3	3033286.7
5	0	0	2	1	3	535443.6	8976.5	2481000.0	8908.8	3034329.0
6	0	0	2	2	3	534053.1	12024.8	2481000.0	9603.4	3036681.3
7	0	2	0	0	3	542783.0	12295.8	248700.0	24684.7	828463.5
8	0	2	0	1	3	542656.8	14071.2	248700.0	24696.2	830124.3
9	0	2	0	2	3	542473.8	19396.7	248700.0	24696.2	835266.7
10	0	2	2	0	3	536498.6	11722.7	2481000.0	26197.1	3055418.4
11	0	2	2	1	3	536138.4	13485.4	2481000.0	26208.6	3056832.5
12	0	2	2	2	3	536013.9	18798.4	2481000.0	26208.6	3062020.9
13	2	0	0	0	3	1071575.1	7533.8	248100.0	8700.5	1335909.3
14	2	0	0	1	3	1072006.8	8614.0	248100.0	9149.7	1337870.5
15	2	0	0	2	3	1068117.2	12029.5	248100.0	9603.4	1337850.1
16	2	0	2	0	3	1072117.9	7523.6	2481000.0	8703.3	3569344.8
17	2	0	2	1	3	1070869.0	8977.2	2481000.0	8908.8	3569755.1

Continued on next page

Table A.14 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
18	2	0	2	2	3	1068102.5	12032.8	2481000.0	9603.4	3570738.7
19	2	2	0	0	3	1085553.5	12303.2	248700.0	24684.7	1371241.4
20	2	2	0	1	3	1085300.9	14070.1	248700.0	24696.2	1372767.2
21	2	2	0	2	3	1084945.4	19402.1	248700.0	24696.2	1377743.7
22	2	2	2	0	3	1072969.1	11716.8	2481000.0	26197.1	3591883.0
23	2	2	2	1	3	1072272.8	13486.0	2481000.0	26208.6	3592967.5
24	2	2	2	2	3	1072027.9	18808.5	2481000.0	26208.6	3598045.1

Table A.15: Cost for the published schedule

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
1	0	0	0	0	1	535989.9	225766.2	266100	0	1027856.1
2	0	0	0	1	1	535989.9	225766.2	266100	0	1027856.1
3	0	0	0	2	1	535989.9	225766.2	266100	0	1027856.1
4	0	0	2	0	1	535989.9	225766.2	2661000	0	3422756.1
5	0	0	2	1	1	535989.9	225766.2	2661000	0	3422756.1
6	0	0	2	2	1	535989.9	225766.2	2661000	0	3422756.1
7	0	2	0	0	1	535989.9	225766.2	266100	0	1027856.1
8	0	2	0	1	1	535989.9	225766.2	266100	0	1027856.1
9	0	2	0	2	1	535989.9	225766.2	266100	0	1027856.1
10	0	2	2	0	1	535989.9	225766.2	2661000	0	3422756.1
11	0	2	2	1	1	535989.9	225766.2	2661000	0	3422756.1
12	0	2	2	2	1	535989.9	225766.2	2661000	0	3422756.1
13	2	0	0	0	1	1071979.8	225766.2	266100	0	1563846.0
14	2	0	0	1	1	1071979.8	225766.2	266100	0	1563846.0
15	2	0	0	2	1	1071979.8	225766.2	266100	0	1563846.0
16	2	0	2	0	1	1071979.8	225766.2	2661000	0	3958746.0
17	2	0	2	1	1	1071979.8	225766.2	2661000	0	3958746.0
18	2	0	2	2	1	1071979.8	225766.2	2661000	0	3958746.0
19	2	2	0	0	1	1071979.8	225766.2	266100	0	1563846.0
20	2	2	0	1	1	1071979.8	225766.2	266100	0	1563846.0
21	2	2	0	2	1	1071979.8	225766.2	266100	0	1563846.0
22	2	2	2	0	1	1071979.8	225766.2	2661000	0	3958746.0
23	2	2	2	1	1	1071979.8	225766.2	2661000	0	3958746.0
24	2	2	2	2	1	1071979.8	225766.2	2661000	0	3958746.0
1	0	0	0	0	2	535989.9	225766.2	266100	0	1027856.1
2	0	0	0	1	2	535989.9	225766.2	266100	0	1027856.1
3	0	0	0	2	2	535989.9	225766.2	266100	0	1027856.1
4	0	0	2	0	2	535989.9	225766.2	2661000	0	3422756.1
5	0	0	2	1	2	535989.9	225766.2	2661000	0	3422756.1
6	0	0	2	2	2	535989.9	225766.2	2661000	0	3422756.1
7	0	2	0	0	2	535989.9	225766.2	266100	0	1027856.1

Continued on next page

Table A.15 – Continued from previous page

Run	Factors				Rep.	Costs				
#	A	B	C	D	#	Fuel	Idle Time	Daily Usage	Spill	Total
8	0	2	0	1	2	535989.9	225766.2	266100	0	1027856.1
9	0	2	0	2	2	535989.9	225766.2	266100	0	1027856.1
10	0	2	2	0	2	535989.9	225766.2	2661000	0	3422756.1
11	0	2	2	1	2	535989.9	225766.2	2661000	0	3422756.1
12	0	2	2	2	2	535989.9	225766.2	2661000	0	3422756.1
13	2	0	0	0	2	1071979.8	225766.2	266100	0	1563846.0
14	2	0	0	1	2	1071979.8	225766.2	266100	0	1563846.0
15	2	0	0	2	2	1071979.8	225766.2	266100	0	1563846.0
16	2	0	2	0	2	1071979.8	225766.2	2661000	0	3958746.0
17	2	0	2	1	2	1071979.8	225766.2	2661000	0	3958746.0
18	2	0	2	2	2	1071979.8	225766.2	2661000	0	3958746.0
19	2	2	0	0	2	1071979.8	225766.2	266100	0	1563846.0
20	2	2	0	1	2	1071979.8	225766.2	266100	0	1563846.0
21	2	2	0	2	2	1071979.8	225766.2	266100	0	1563846.0
22	2	2	2	0	2	1071979.8	225766.2	2661000	0	3958746.0
23	2	2	2	1	2	1071979.8	225766.2	2661000	0	3958746.0
24	2	2	2	2	2	1071979.8	225766.2	2661000	0	3958746.0
1	0	0	0	0	3	535989.9	225766.2	266100	0	1027856.1
2	0	0	0	1	3	535989.9	225766.2	266100	0	1027856.1
3	0	0	0	2	3	535989.9	225766.2	266100	0	1027856.1
4	0	0	2	0	3	535989.9	225766.2	2661000	0	3422756.1
5	0	0	2	1	3	535989.9	225766.2	2661000	0	3422756.1
6	0	0	2	2	3	535989.9	225766.2	2661000	0	3422756.1
7	0	2	0	0	3	535989.9	225766.2	266100	0	1027856.1
8	0	2	0	1	3	535989.9	225766.2	266100	0	1027856.1
9	0	2	0	2	3	535989.9	225766.2	266100	0	1027856.1
10	0	2	2	0	3	535989.9	225766.2	2661000	0	3422756.1
11	0	2	2	1	3	535989.9	225766.2	2661000	0	3422756.1
12	0	2	2	2	3	535989.9	225766.2	2661000	0	3422756.1
13	2	0	0	0	3	1071979.8	225766.2	266100	0	1563846.0
14	2	0	0	1	3	1071979.8	225766.2	266100	0	1563846.0
15	2	0	0	2	3	1071979.8	225766.2	266100	0	1563846.0
16	2	0	2	0	3	1071979.8	225766.2	2661000	0	3958746.0
17	2	0	2	1	3	1071979.8	225766.2	2661000	0	3958746.0
18	2	0	2	2	3	1071979.8	225766.2	2661000	0	3958746.0
19	2	2	0	0	3	1071979.8	225766.2	266100	0	1563846.0
20	2	2	0	1	3	1071979.8	225766.2	266100	0	1563846.0
21	2	2	0	2	3	1071979.8	225766.2	266100	0	1563846.0
22	2	2	2	0	3	1071979.8	225766.2	2661000	0	3958746.0
23	2	2	2	1	3	1071979.8	225766.2	2661000	0	3958746.0
24	2	2	2	2	3	1071979.8	225766.2	2661000	0	3958746.0

Table A.16: CPU time

Run	Factors				Rep.	CPU Time in sec.	
#	A	B	C	D	#	Heuristic1	Heuristic1
1	0	0	0	0	1	5400	3683.03
2	0	0	0	1	1	5400	3783.41
3	0	0	0	2	1	5400	5400.00
4	0	0	2	0	1	5400	4441.16
5	0	0	2	1	1	5400	3959.49
6	0	0	2	2	1	5400	3792.68
7	0	2	0	0	1	5400	2371.01
8	0	2	0	1	1	5400	2748.85
9	0	2	0	2	1	5400	3874.28
10	0	2	2	0	1	5400	2422.40
11	0	2	2	1	1	5400	2646.12
12	0	2	2	2	1	5400	3332.23
13	2	0	0	0	1	5400	3713.88
14	2	0	0	1	1	5400	4342.55
15	2	0	0	2	1	5400	5400.00
16	2	0	2	0	1	5400	5126.85
17	2	0	2	1	1	5400	4416.34
18	2	0	2	2	1	5400	3631.56
19	2	2	0	0	1	5400	2426.86
20	2	2	0	1	1	5400	2702.25
21	2	2	0	2	1	5400	3806.13
22	2	2	2	0	1	5400	2394.51
23	2	2	2	1	1	5400	2539.31
24	2	2	2	2	1	5400	3353.52
1	0	0	0	0	2	5400	5037.63
2	0	0	0	1	2	5400	4052.59
3	0	0	0	2	2	5400	4557.12
4	0	0	2	0	2	5400	5400.00
5	0	0	2	1	2	5400	3766.53
6	0	0	2	2	2	5400	3452.46
7	0	2	0	0	2	5400	2705.57
8	0	2	0	1	2	5400	3465.42
9	0	2	0	2	2	5400	2643.73
10	0	2	2	0	2	5400	4168.83
11	0	2	2	1	2	5400	3481.74
12	0	2	2	2	2	5400	2698.72
13	2	0	0	0	2	5400	4667.22
14	2	0	0	1	2	5400	4281.32
15	2	0	0	2	2	5400	4455.03
16	2	0	2	0	2	5400	5400.00
17	2	0	2	1	2	5400	3562.13
18	2	0	2	2	2	5400	3501.02
19	2	2	0	0	2	5400	2778.86
20	2	2	0	1	2	5400	3168.49
21	2	2	0	2	2	5400	2636.35

*Continued on next page*

Table A.16 – *Continued from previous page*

Run	Factors				Rep.	CPU Time in sec.	
#	A	B	C	D	#	Heuristic1	Heuristic1
22	2	2	2	0	2	5400	3951.33
23	2	2	2	1	2	5400	3478.04
24	2	2	2	2	2	5400	2659.69
1	0	0	0	0	3	5400	2516.06
2	0	0	0	1	3	5400	2128.90
3	0	0	0	2	3	5400	2372.82
4	0	0	2	0	3	5400	2342.34
5	0	0	2	1	3	5400	2246.31
6	0	0	2	2	3	5400	2724.39
7	0	2	0	0	3	5400	1902.17
8	0	2	0	1	3	1034.29	2025.30
9	0	2	0	2	3	3352.18	3737.97
10	0	2	2	0	3	2224.89	2069.87
11	0	2	2	1	3	724.84	2251.69
12	0	2	2	2	3	3346.85	2485.38
13	2	0	0	0	3	5400	2636.51
14	2	0	0	1	3	5400	2196.49
15	2	0	0	2	3	5400	2956.80
16	2	0	2	0	3	5400	2300.92
17	2	0	2	1	3	5400	2201.66
18	2	0	2	2	3	5400	2693.90
19	2	2	0	0	3	5400	1981.54
20	2	2	0	1	3	5400	1987.59
21	2	2	0	2	3	5400	3653.92
22	2	2	2	0	3	5400	1702.56
23	2	2	2	1	3	5400	2234.22
24	2	2	2	2	3	5400	2684.12



# Vita

Hüseyin Gürkan was born on April 26, 1989 in Denizli, Turkey. He graduated from Denizli Science High School in 2007. In 2012, he earned a B.S. in industrial engineering from Bilkent University and proceeded to graduate study in M.S. program of the same department. Since then, he has been working with Prof. M. Selim Aktürk. He had been on the grant 2210 awarded by The Scientific and Technological Research Council of Turkey (TUBITAK).